Estimation of the spectral envelope in the LSP-HS space by matrix transforms

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Abstract

The coding theory of the forms of spectral envelope of speech signals of the using linear spectral parameters of the highest splitting (LSP-HS) is regarded. Advantages of using of LSP-HS parameters compare with classic LSP parameters are specified. It is demonstrated that the direct and inverse transform of LSP-HS method can be regarded as a certain matrices transform from the LPC coefficients. The formula of estimates of the spectral envelope of the synthesized speech signal in space LSP-HS is written.

Keywords: Encoding shape of the spectral envelope, linear spectral parameters (LSP), linear spectral pairs (projection LSPr), linear spectral frequencies (LSF), linear prediction coefficients (LPC).

1. Introduction

The new original method of encoding of spectral envelope of speech signal (that had been named as LSP-HS — linear spectral parameter of the highest splitting) has been described in previous works [1 — 7]. This method is recommended to using in the speech transformation devices of the receiving and transmitting equipments, which are based on the linear prediction (LP) algorithms. The main idea of method is in that characteristic polynomial $A(z)$ of prediction filter of M-orded, which represented as one stable polynomial of M-order,

$$A(z) = 1 - \sum_{i=1}^{M} a_i' z^{-i} = 1 + \sum_{i=1}^{M} a_i z^{-i}, \qquad (1)$$

is proposed to be represented as M elementary stable normalized polynomials of 1th order

$$A^{vvvv}(z) = 1 + a_1^{vvvv} z^{-1}, \qquad (2)$$

which are the results of step-by-step splitting of original polynomial $A(z)$, fig. 1.

The roots of elementary polynomials (2), $A^{vvvv}(z) = 1 + a_1^{vvvv} z^{-1}$, are the linear spectral *projections* of the highest splitting (LSP-HS).

Arccosine from roots of elementary polynomials (2), $A^{vvvv}(z) = 1 + a_1^{vvvv} z^{-1}$, are the linear spectral *frequencies* of the highest splitting (LSF-HS).

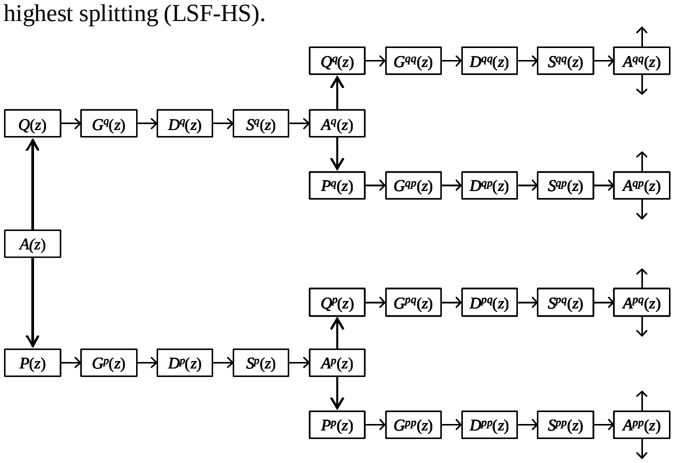

Fig. 1. Structure of LSP-HS inverse transform for stages from 1 to 2

2. Goals of LSPr-HS and LSF-HS compared with classic LSP

It was shown that the classical method of linear spectral parameters (frequencies and pairs — LSF, LSPr) are to be only the first stage of regression of the method of linear spectral parameters (projection and frequencies) of the highest splitting — LSF-HS, LSPr-HS.

Transition from classical (first stage of splitting) LSP to the LSP-HS allow retaining the advantage of the classical method and simultaneously obtaining a number of goals:

1. The process of representing of prediction filter (1), $A(z) = 1 - \sum_{i=1}^{M} a_i' z^{-i} = 1 + \sum_{i=1}^{M} a_i z^{-i}$, by LSP-HS is simplified and acquires strict and logically finished form. Roots of elementary polynomials (2), $A^{vvvv}(z) = 1 + a_1^{vvvv} z^{-1}$, are calculated trivially without using the iterative estimation method since they are equal to the coefficients a_i^{vvvv} with respect to the sign. Elementary polynomials in case they are obtained at early stages of splitting remains invariant with respect to the further stages of splitting and do not depend from the value of M in (1), $A(z) = 1 - \sum_{i=1}^{M} a_i' z^{-i} = 1 + \sum_{i=1}^{M} a_i z^{-i}$, [1, 2, 7].

2. Methodological estimation error of the linear spectral parameters, peculiar to the classical method and appearing as a result of iteration search of the real interleaving roots of polynomials pair $D^p(x)$ and $D^q(x)$, which in case of the 10th order linear prediction have the form $D^v(x) = x^5 + d_1^v x^4 + d_2^v x^3 + d_3^v x^2 + d_4^v x + d_5^v$ (here the v means any p or q symbol) is eliminated [1, 2].

3. The algorithm of representing the linear prediction coefficients (LPC) in terms of LSP is accelerated [3, 4].

4. The required computational power is distributed between the analyzer of the transmitting side and synthesizer on the receiver side of the speech transformation device more uniformly [5, 6].

5. There exists a simple encoding rule fore the chain of upper symbol indexes of coefficients s_i^{vvvv} (where $s_i^{vvvv} \equiv a_i^{vvvv}$), that resembling the history of forming the coefficients in the process stage-wise splitting from (1), $A(z) = 1 - \sum_{i=1}^{M} a_i' z^{-i} = 1 + \sum_{i=1}^{M} a_i z^{-i}$, to (2), $A^{vvvv}(z) = 1 + a_1^{vvvv} z^{-1}$, which allow making transition to the numerical indexes of coefficients $s_1,\ldots,s_M$, and back to the chain of the upper symbol indexes [7]. Numerical indexes allow plotting the graph of coefficients of LSP-HS, fig. 2, and determining the getting of elementary invariant normalized stable 1th degree polynomials (2), $A^{vvvv}(z) = 1 + a_1^{vvvv} z^{-1}$, at early stages of splitting for an arbitrary value of M in (1), $A(z) = 1 - \sum_{i=1}^{M} a_i' z^{-i} = 1 + \sum_{i=1}^{M} a_i z^{-i}$, [7], additionally decreasing the number of performed operation.

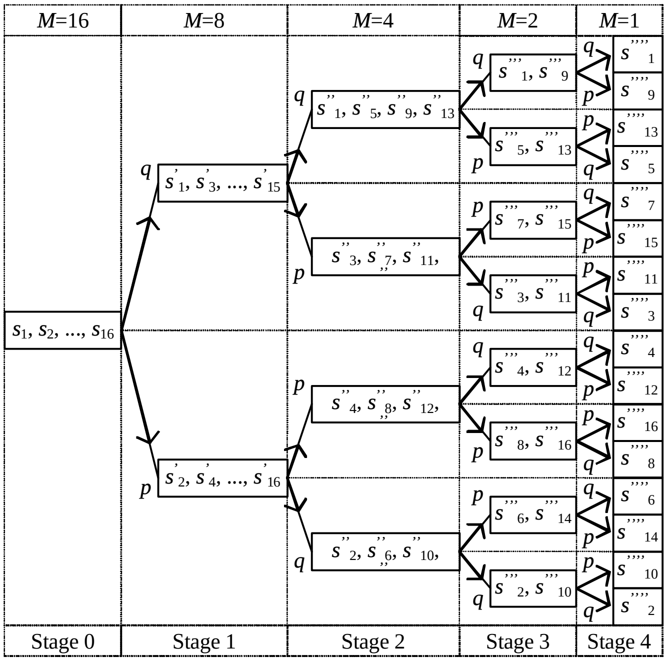

Fig. 2. Graph of formation of LSP-HS coefficients for $M = 16$ (it is shown how to changes value of degree M for each stages)

6. There is a simple stability criterion for the synthesized filter in terms of LSP-HS with numerical indexes in coefficients $s_1,\ldots,s_M$, which is invariant for any M, [7]:

$$-1 < s_1 < s_2 < s_3 < \ldots < s_M < 1.$$

7. LSP-HS provides a less error when executing inter-frame interpolation compared to other equivalent parameters, including classical (first stage splitting) LSP, [8].

8. LSP-HS provides a less error vector quantization compared to other equivalent parameters, including classical (first stage splitting) LSP, [9].

9. LSP-HS provides lower prediction error of the shape of spectral envelope of the speech signal on the basis of the known values at the previous frame, [10].

3. LSP-HS inverse transform

All operations in the inverse transformation of LSP-HS method are linear and LPC, which are recovered from LSP-HS coefficients as a result of the inverse transformation can be determined by using a generalized linear function of 10 variables: $a_i = f_i(s_1, s_2, \ldots, s_{10}) = f_{i,0} + f_{i,1}s_1 + f_{i,2}s_2 + \ldots + f_{i,10}s_{10}$, where $s_1 = s_1^{qqqq}$, $s_2 = s_1^{pqqq}$, $s_3 = s_1^{qpqq}$, …, $s_{10} = s_1^{pqqp}$ — ordered LSP-HS coefficients with index numbers, which correspond to the numbering, that is started from the one. These coefficients satisfy the rule:

$$-1 < s_1 < s_2 < s_3 < s_4 < s_5 < s_6 < s_7 < s_8 < s_9 < s_{10} < +1. \quad (3)$$

Digital indexation of the LSP-HS coefficients are clearly related to their symbolic indexing, which reflects the history of the step-by-step splitting of polynomials in the direct transform of LSP-HS method. This relationship can be defined by the rule:

1. Each character index q and p is associated with a logical zero and a logical one (q=0, p=1).
2. Chain of indices q и p, that reflect the history of the formation of the coefficients resulting from the step-by-step splitting of polynomials in the direct transform of LSP-HS method, is seen as a binary code with the weight of each bit: the weight of the first stage — 2^0, the weight of the second stage — 2^1, and so on.
3. For convenience, each digital index, which is calculated by definition binary code is increased by one, what producing the numbering of the coefficients, that starts from the one (as opposed to indexing that starts with zero).

An example of a specified value for the first 16 numbers of coefficients are given in Table 1.

Said function can be rewritten in a more compact form: $a_i = \sum_{j=0}^{M_A} f_{i,j}s_j$, where $s_0 \equiv 1$ — formal parameter, which is introduced for convenience, $M_A = 10$ — order of LP and degree of polynomial $A(z) = 1 + \sum_{i=1}^{M_A} a_i z^{-i}$.

Table 1. Relationship of character and numeric indices for $M_A = 16$

Chain of character indices	Chain of binary indices with weights $2^0 2^1 2^2 2^3$	Digital indices, starts from 0	Number, starts from 1	Chain of character indices	Chain of binary indices with weights $2^0 2^1 2^2 2^3$	Digital indices, starts from 0	Number, starts from 1
qqqq	0000	0	1	*pqqq*	1000	1	2
qqqp	0001	8	9	*pqqp*	1001	9	10
qqpq	0010	4	5	*pqpq*	1010	5	6
qqpp	0011	12	13	*pqpp*	1011	13	14
qpqq	0100	2	3	*ppqq*	1100	3	4
qpqp	0101	10	11	*ppqp*	1101	11	12
qppq	0110	6	7	*pppq*	1110	7	8
qppp	0111	14	15	*pppp*	1111	15	16

The coefficients a_i of polynomial

$$A(z) = 1 + \sum_{i=1}^{M_A} a_i z^{-i} \quad (4)$$

can be combined into a vector $\mathbf{A} = [1 \;\; a_1 \;\; a_2 \;\; \ldots \;\; a_{10}]^{\mathrm{T}}$, and the value of the complex variable z^{-i}, $0 \le i \le 10$, — into a vector $\mathbf{Z} = [1 \;\; z^{-1} \;\; z^{-2} \;\; \ldots \;\; z^{-10}]^{\mathrm{T}}$. Then

$$A(z) = \mathbf{A}^{\mathrm{T}}\mathbf{Z} = \mathbf{Z}^{\mathrm{T}}\mathbf{A}. \quad (5)$$

By analogy, the result of direct transformation of LPC in LSP-HS space can be combined into a LSP-HS vector $\mathbf{S} = [1 \;\; s_1 \;\; s_2 \;\; \ldots \;\; s_{10}]^{\mathrm{T}}$. By the linear relationship $a_i = f_i(s_1, s_2, \ldots, s_{10})$, $0 \le i \le 10$, we can write the 10 equations

$$\begin{cases} a_1 = f_1(s_1, s_2, \ldots, s_{10}) \\ a_2 = f_2(s_1, s_2, \ldots, s_{10}) \\ \ldots \\ a_{10} = f_{10}(s_1, s_2, \ldots, s_{10}) \end{cases} \text{ as } \begin{cases} f_{1,0} + f_{1,1}s_1 + f_{1,2}s_2 + \ldots + f_{1,10}s_{10} = a_1 \\ f_{2,0} + f_{2,1}s_1 + f_{2,2}s_2 + \ldots + f_{2,10}s_{10} = a_2 \\ \ldots \\ f_{10,0} + f_{10,1}s_1 + f_{10,2}s_2 + \ldots + f_{10,10}s_{10} = a_{10} \end{cases}.$$

The 11[th] trivial equation can be added to bring the system to a quadratic form:

$$\begin{cases} 1 + 0 \cdot s_1 + 0 \cdot s_2 + \ldots + 0 \cdot s_{10} = 1 \\ f_{1,0} + f_{1,1}s_1 + f_{1,2}s_2 + \ldots + f_{1,10}s_{10} = a_1 \\ f_{2,0} + f_{2,1}s_1 + f_{2,2}s_2 + \ldots + f_{2,10}s_{10} = a_2 \\ \ldots \\ f_{10,0} + f_{10,1}s_1 + f_{10,2}s_2 + \ldots + f_{10,10}s_{10} = a_{10} \end{cases}$$

, which can be rewrote in a matrix form

$$\begin{bmatrix} f_{0,0} & f_{0,1} & f_{0,2} & \cdots & f_{0,10} \\ f_{1,0} & f_{1,1} & f_{1,2} & \cdots & f_{1,10} \\ f_{2,0} & f_{2,1} & f_{2,2} & \cdots & f_{2,10} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ f_{10,0} & f_{10,1} & f_{10,2} & \cdots & f_{10,10} \end{bmatrix} \begin{bmatrix} 1 \\ s_1 \\ s_2 \\ \cdots \\ s_{10} \end{bmatrix} = \begin{bmatrix} 1 \\ a_1 \\ a_2 \\ \cdots \\ a_{10} \end{bmatrix} \text{ or}$$

$$\mathbf{FS}=\mathbf{A}\ ,\ \mathbf{F}=\begin{bmatrix} f_{0,0} & f_{0,1} & f_{0,2} & \cdots & f_{0,10} \\ f_{1,0} & f_{1,1} & f_{1,2} & \cdots & f_{1,10} \\ f_{2,0} & f_{2,1} & f_{2,2} & \cdots & f_{2,10} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ f_{10,0} & f_{10,1} & f_{10,2} & \cdots & f_{10,10} \end{bmatrix},\ \begin{matrix} f_{0,0}=1 \\ f_{0,i}=0 \\ 1\le i\le 10 \end{matrix}\ . \qquad (6)$$

Abstracting from the stability criteria in terms synthesizer filter LSP-HP and rule (3), and acting more formal, we can set 11 test vectors LSP-HS: $\mathbf{S}_0=\begin{bmatrix}1\\0\\0\\ \\ \cdot\\0\end{bmatrix}$, $\mathbf{S}_1=\begin{bmatrix}1\\1\\0\\ \\ \cdot\\0\end{bmatrix}$, $\mathbf{S}_2=\begin{bmatrix}1\\0\\1\\ \\ \cdot\\0\end{bmatrix}$, …, $\mathbf{S}_{10}=\begin{bmatrix}1\\0\\0\\ \\ \cdot\\1\end{bmatrix}$, which are the columns of the matrix $\tilde{\mathbf{E}}=\begin{bmatrix}1 & 1 & 1 & \cdots & 1\\ 0 & 1 & 0 & \cdots & 0\\ 0 & 0 & 1 & \cdots & 0\\ \cdot & \cdots & \cdots & \cdots & \cdots\\ 0 & 0 & 0 & \cdots & 1\end{bmatrix}$.

By using algorithm of inverse transform from LSP-HS into LPC, $\mathbf{A}=\mathrm{rpt}(\mathbf{S})$, for each testing vectors $\mathbf{S}_0$, $\mathbf{S}_1$, $\mathbf{S}_2$, …, $\mathbf{S}_{10}$ we can find 11 corresponding result LPC vectors $\mathbf{A}_0$, $\mathbf{A}_1$, $\mathbf{A}_2$, …, $\mathbf{A}_{10}$, which must satisfy the matrix equation (6), $\mathbf{FS}=\mathbf{A}$. Then we can write:

$$\mathbf{A}_0=\begin{bmatrix}1\\a_{0,1}\\a_{0,2}\\ \cdots\\a_{0,10}\end{bmatrix}=\begin{bmatrix} f_{0,0} & f_{0,1} & f_{0,2} & \cdots & f_{0,10} \\ f_{1,0} & f_{1,1} & f_{1,2} & \cdots & f_{1,10} \\ f_{2,0} & f_{2,1} & f_{2,2} & \cdots & f_{2,10} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ f_{10,0} & f_{10,1} & f_{10,2} & \cdots & f_{10,10} \end{bmatrix}\begin{bmatrix}1\\0\\0\\ \\ \cdot\\0\end{bmatrix},\ \mathbf{A}_1=\begin{bmatrix}1\\a_{1,1}\\a_{1,2}\\ \cdots\\a_{1,10}\end{bmatrix}=$$

$$\begin{bmatrix} f_{0,0} & f_{0,1} & f_{0,2} & \cdots & f_{0,10} \\ f_{1,0} & f_{1,1} & f_{1,2} & \cdots & f_{1,10} \\ f_{2,0} & f_{2,1} & f_{2,2} & \cdots & f_{2,10} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ f_{10,0} & f_{10,1} & f_{10,2} & \cdots & f_{10,10} \end{bmatrix}\begin{bmatrix}1\\1\\0\\ \\ \cdot\\0\end{bmatrix},$$

$$\mathbf{A}_2=\begin{bmatrix}1\\a_{2,1}\\a_{2,2}\\ \cdots\\a_{2,10}\end{bmatrix}=\begin{bmatrix} f_{0,0} & f_{0,1} & f_{0,2} & \cdots & f_{0,10} \\ f_{1,0} & f_{1,1} & f_{1,2} & \cdots & f_{1,10} \\ f_{2,0} & f_{2,1} & f_{2,2} & \cdots & f_{2,10} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ f_{10,0} & f_{10,1} & f_{10,2} & \cdots & f_{10,10} \end{bmatrix}\begin{bmatrix}1\\0\\1\\ \\ \cdot\\0\end{bmatrix},\ldots,$$

$$\mathbf{A}_{10}=\begin{bmatrix}1\\a_{10,1}\\a_{10,2}\\ \cdots\\a_{10,10}\end{bmatrix}=\begin{bmatrix} f_{0,0} & f_{0,1} & f_{0,2} & \cdots & f_{0,10} \\ f_{1,0} & f_{1,1} & f_{1,2} & \cdots & f_{1,10} \\ f_{2,0} & f_{2,1} & f_{2,2} & \cdots & f_{2,10} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ f_{10,0} & f_{10,1} & f_{10,2} & \cdots & f_{10,10} \end{bmatrix}\begin{bmatrix}1\\0\\0\\ \\ \cdot\\1\end{bmatrix},$$

that can be combined into a single matrix equation

$$\begin{bmatrix} f_{0,0} & f_{0,1} & f_{0,2} & \cdots & f_{0,10} \\ f_{1,0} & f_{1,1} & f_{1,2} & \cdots & f_{1,10} \\ f_{2,0} & f_{2,1} & f_{2,2} & \cdots & f_{2,10} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ f_{10,0} & f_{10,1} & f_{10,2} & \cdots & f_{10,10} \end{bmatrix}\begin{bmatrix}1 & 1 & 1 & \cdots & 1\\ 0 & 1 & 0 & \cdots & 0\\ 0 & 0 & 1 & \cdots & 0\\ \cdot & \cdots & \cdots & \cdots & \cdots\\ 0 & 0 & 0 & \cdots & 1\end{bmatrix}=$$

$$=\begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ a_{0,1} & a_{1,1} & a_{2,1} & \cdots & a_{10,1} \\ a_{0,2} & a_{1,2} & a_{2,2} & \cdots & a_{10,2} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{0,10} & a_{1,10} & a_{2,10} & \cdots & a_{10,10} \end{bmatrix}. \qquad (7)$$

Matrix $\tilde{\mathbf{E}}=\begin{bmatrix}1 & 1 & 1 & \cdots & 1\\ 0 & 1 & 0 & \cdots & 0\\ 0 & 0 & 1 & \cdots & 0\\ \cdot & \cdots & \cdots & \cdots & \cdots\\ 0 & 0 & 0 & \cdots & 1\end{bmatrix}$ in the resulting matrix equation can be reduced to a single form. To do this, each of the columns, except the first should take it away first column. In order matrix equation is not disrupted, similar operations to execute and with columns of the matrix, which is the right side of the matrix equation (7). Then we can get:

$$\begin{bmatrix} f_{0,0} & f_{0,1} & f_{0,2} & \cdots & f_{0,10} \\ f_{1,0} & f_{1,1} & f_{1,2} & \cdots & f_{1,10} \\ f_{2,0} & f_{2,1} & f_{2,2} & \cdots & f_{2,10} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ f_{10,0} & f_{10,1} & f_{10,2} & \cdots & f_{10,10} \end{bmatrix}\begin{bmatrix}1 & 0 & 0 & \cdots & 0\\ 0 & 1 & 0 & \cdots & 0\\ 0 & 0 & 1 & \cdots & 0\\ \cdot & \cdots & \cdots & \cdots & \cdots\\ 0 & 0 & 0 & \cdots & 1\end{bmatrix}=$$

$$\begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ a_{0,1} & a_{1,1}-a_{0,1} & a_{2,1}-a_{0,1} & \cdots & a_{10,1}-a_{0,1} \\ a_{0,2} & a_{1,2}-a_{0,2} & a_{2,2}-a_{0,2} & \cdots & a_{10,2}-a_{0,2} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{0,10} & a_{1,10}-a_{0,10} & a_{2,10}-a_{0,10} & \cdots & a_{10,10}-a_{0,10} \end{bmatrix}.$$

Finally, the matrix inverse transform LSP-HS $\mathbf{F}$, is defined through the components of the resulting LPC vector is such:

$$\mathbf{F}=\begin{bmatrix} f_{0,0} & f_{0,1} & f_{0,2} & \cdots & f_{0,10} \\ f_{1,0} & f_{1,1} & f_{1,2} & \cdots & f_{1,10} \\ f_{2,0} & f_{2,1} & f_{2,2} & \cdots & f_{2,10} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ f_{10,0} & f_{10,1} & f_{10,2} & \cdots & f_{10,10} \end{bmatrix}=$$

$$\begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ a_{0,1} & a_{1,1}-a_{0,1} & a_{2,1}-a_{0,1} & \cdots & a_{10,1}-a_{0,1} \\ a_{0,2} & a_{1,2}-a_{0,2} & a_{2,2}-a_{0,2} & \cdots & a_{10,2}-a_{0,2} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{0,10} & a_{1,10}-a_{0,10} & a_{2,10}-a_{0,10} & \cdots & a_{10,10}-a_{0,10} \end{bmatrix}.$$

Value of the matrix of inverse transform LSP-HS $\mathbf{F}$, that have been obtained from the above methodic for $M_A=10$ are next:

F = (8)

1	0	0	0	0	0	0	0	0	0	0
0	1	1	1	1	1	1	1	1	1	1
45	9	7	5	3	1	-1	-3	-5	-7	-9
0	36	20	8	0	-4	-4	0	8	20	36
210	84	28	0	-8	-4	4	8	0	-28	-84
0	126	14	-14	-6	6	6	-6	-14	14	126
210	126	-14	-14	6	6	-6	-6	14	14	-126
0	84	-28	0	8	-4	-4	8	0	-28	84
45	36	-20	8	0	-4	4	0	-8	20	-36
0	9	-7	5	-3	1	1	-3	5	-7	9
1	1	-1	1	-1	1	-1	1	-1	1	-1

4. LSP-HS direct transform

Similarly, the all operation of the direct transform of LPC into LSP-HS can be analyzed and can be shown that direct transformation $\mathbf{S} = \mathrm{dpt}(\mathbf{A})$ is also a linear. Then, the coefficients of LSP-HS can be determined by using a generalized linear function of 10 variables: $s_i = \varphi_i(a_1, a_2, ..., a_{10}) = \varphi_{i,0} + \varphi_{i,1} a_1 + \varphi_{i,2} a_2 + ... + \varphi_{i,10} a_{10}$, that produce the matrix equation

$$\begin{bmatrix} \varphi_{0,0} & \varphi_{0,1} & \varphi_{0,2} & \cdots & \varphi_{0,10} \\ \varphi_{1,0} & \varphi_{1,1} & \varphi_{1,2} & \cdots & \varphi_{1,10} \\ \varphi_{2,0} & \varphi_{2,1} & \varphi_{2,2} & \cdots & \varphi_{2,10} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \varphi_{10,0} & \varphi_{10,1} & \varphi_{10,2} & \cdots & \varphi_{10,10} \end{bmatrix} \begin{bmatrix} 1 \\ a_1 \\ a_2 \\ \cdots \\ a_{10} \end{bmatrix} = \begin{bmatrix} 1 \\ s_1 \\ s_2 \\ \cdots \\ s_{10} \end{bmatrix} \text{ or}$$

$$\mathbf{\Phi A} = \mathbf{S}, \text{ where } \mathbf{\Phi} = \begin{bmatrix} \varphi_{0,0} & \varphi_{0,1} & \varphi_{0,2} & \cdots & \varphi_{0,10} \\ \varphi_{1,0} & \varphi_{1,1} & \varphi_{1,2} & \cdots & \varphi_{1,10} \\ \varphi_{2,0} & \varphi_{2,1} & \varphi_{2,2} & \cdots & \varphi_{2,10} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \varphi_{10,0} & \varphi_{10,1} & \varphi_{10,2} & \cdots & \varphi_{10,10} \end{bmatrix}, \begin{matrix} \varphi_{0,0} = 1 \\ \varphi_{0,i} = 0 \\ 1 \le i \le 10 \end{matrix} \quad (9)$$

Solving the equation in a manner similar to that described above, you can get the matrix of direct conversion of LSP-HS $\mathbf{\Phi}$, defined through the components of the resulting LSP-HS vector, which is the:

$$\mathbf{\Phi} = \begin{bmatrix} \varphi_{0,0} & \varphi_{0,1} & \varphi_{0,2} & \cdots & \varphi_{0,10} \\ \varphi_{1,0} & \varphi_{1,1} & \varphi_{1,2} & \cdots & \varphi_{1,10} \\ \varphi_{2,0} & \varphi_{2,1} & \varphi_{2,2} & \cdots & \varphi_{2,10} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \varphi_{10,0} & \varphi_{10,1} & \varphi_{10,2} & \cdots & \varphi_{10,10} \end{bmatrix} =$$

$$\begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ s_{0,1} & s_{1,1} - s_{0,1} & s_{2,1} - s_{0,1} & \cdots & s_{10,1} - s_{0,1} \\ s_{0,2} & s_{1,2} - s_{0,2} & s_{2,2} - s_{0,2} & \cdots & s_{10,2} - s_{0,2} \\ .. & \cdots & \cdots & \cdots & \cdots \\ s_{0,10} & s_{1,10} - s_{0,10} & s_{2,10} - s_{0,10} & \cdots & s_{10,10} - s_{0,10} \end{bmatrix}.$$

The value of the matrix elements of direct transform of LSP-HS $\mathbf{\Phi}$, which were obtained by computing the above calculations for $M_A = 10$ are next:

$\mathbf{\Phi} =$

1	0	0	0	0	0	0	0	0	0	0
-0.998046875	0.001953125	0.001953125	0.001953125	0.001953125	0.001953125	0.001953125	0.001953125	0.001953125	0.001953125	0.001953125
-0.978515625	0.017578125	0.013671875	0.009765625	0.005859375	0.001953125	-0.001953125	-0.005859375	-0.009765625	-0.013671875	-0.017578125
-0.890625000	0.070312500	0.039062500	0.015625000	0.000000000	-0.007812500	-0.007812500	0.000000000	0.015625000	0.039062500	0.070312500
-0.656250000	0.164062500	0.054687500	0.000000000	-0.015625000	-0.007812500	0.007812500	0.015625000	0.000000000	-0.054687500	-0.164062500
-0.246093750	0.246093750	0.027343750	-0.027343750	-0.011718750	0.011718750	0.011718750	-0.011718750	-0.027343750	0.027343750	0.246093750
0.246093750	0.246093750	-0.027343750	-0.027343750	0.011718750	0.011718750	-0.011718750	-0.011718750	0.027343750	0.027343750	-0.246093750
0.656250000	0.164062500	-0.054687500	0.000000000	0.015625000	-0.007812500	-0.007812500	0.015625000	0.000000000	-0.054687500	0.164062500
0.890625000	0.070312500	-0.039062500	0.015625000	0.000000000	-0.007812500	0.007812500	0.000000000	-0.015625000	0.039062500	-0.070312500
0.978515625	0.017578125	-0.013671875	0.009765625	-0.005859375	0.001953125	0.001953125	-0.005859375	0.009765625	-0.013671875	0.017578125
0.998046875	0.001953125	-0.001953125	0.001953125	-0.001953125	0.001953125	-0.001953125	0.001953125	-0.001953125	0.001953125	-0.001953125

(10)

5. Basis vectors of the matrix LSP-HS method

Matrix of direct (10) and inverse (8) transform of LSP-HS method can be calculated for any degree M_A. Direct and inverse matrix LSP-HS transformation can be seen as expansion in the corresponding basis vectors.

View of the basic functions of direct and inverse matrix LSP-HS transformation for $M_A = 10$ is shown in Figure. 3 and Figure. 4.

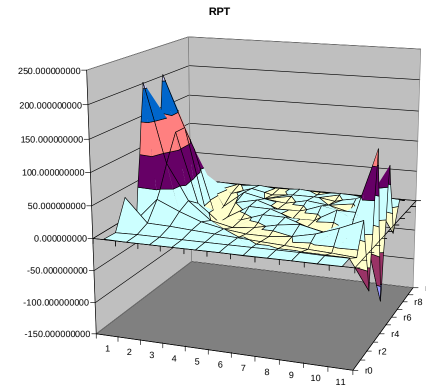

Fig. 3. Basic vectors of the inverse matrix LSP-HS transformation for $M_A = 10$

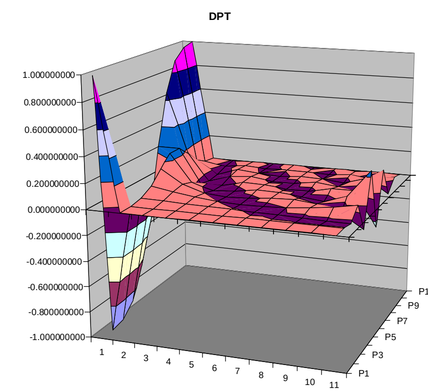

Fig. 4. Basic vectors of the direct matrix LSP-HS transformation for $M_A = 10$

6. Estimation of the spectral envelope in the LSP-HS space

Substituting (6) into (5) we can get the expression for computing polynomial (4) through the coefficients of direct LSP-HS transformation:

$$A(z) = \mathbf{A}^{\mathrm{T}}\mathbf{Z} = \mathbf{Z}^{\mathrm{T}}\mathbf{A} = (\mathbf{FS})^{\mathrm{T}}\mathbf{Z} = \mathbf{S}^{\mathrm{T}}(\mathbf{F}^{\mathrm{T}}\mathbf{Z}) = (\mathbf{Z}^{\mathrm{T}}\mathbf{F})\mathbf{S}, \quad (11)$$

which can be rewritten as

$$A(z) = \mathbf{S}^{\mathrm{T}}\tilde{\mathbf{Z}} = \tilde{\mathbf{Z}}^{\mathrm{T}}\mathbf{S}, \quad (12)$$

where

$$\tilde{\mathbf{Z}} = (\mathbf{F}^{\mathrm{T}}\mathbf{Z}) = \begin{bmatrix} f_{0,0} & f_{0,1} & f_{0,2} & \cdots & f_{0,10} \\ f_{1,0} & f_{1,1} & f_{1,2} & \cdots & f_{1,10} \\ f_{2,0} & f_{2,1} & f_{2,2} & \cdots & f_{2,10} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ f_{10,0} & f_{10,1} & f_{10,2} & \cdots & f_{10,10} \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} 1 \\ z^{-1} \\ z^{-2} \\ \cdots \\ z^{-10} \end{bmatrix}, \quad (13)$$

is a vector, rotate the directions of basis vectors of matrix LSP-HS inverse transform $\mathbf{F}$. Using the substitution $z = e^{j\phi} = e^{j2\pi\omega/\omega_s}$, where ω_s — circular sampling rate, we can be obtained spectral envelope of the synthesized speech signal for a given frequency ω, $K(j\omega) = \left.\frac{1}{A(z)}\right|_{z=e^{j2\pi\omega/\omega_s}}$:

$$K(j\omega)=\left(\begin{bmatrix}1\\ s_1\\ s_2\\ \cdots\\ s_{10}\end{bmatrix}^{T}\left(\begin{bmatrix} f_{0,0} & f_{0,1} & f_{0,2} & \cdots & f_{0,10}\\ f_{1,0} & f_{1,1} & f_{1,2} & \cdots & f_{1,10}\\ f_{2,0} & f_{2,1} & f_{2,2} & \cdots & f_{2,10}\\ \cdots & \cdots & \cdots & \cdots & \cdots\\ f_{10,0} & f_{10,1} & f_{10,2} & \cdots & f_{10,10}\end{bmatrix}^{T}\begin{bmatrix}1\\ e^{-j2\pi\omega/\omega_s}\\ e^{-j4\pi\omega/\omega_s}\\ \cdots\\ e^{-j20\pi\omega/\omega_s}\end{bmatrix}\right)\right)^{-1} \quad (14)$$

7. Extractions

It is shown that the direct and inverse transform of LSP-HS method can be regarded as a certain matrix transformation of coefficients of the polynomial (4).

The method for determining the matrix of direct and inverse LSP-HS matrix transformations is described.

Matrix of direct and inverse LSP-HS transform can be calculated for any degree M_A of polynomial (4).

Matrix of direct (10) and inverse (8) LSP-HS transformation for $M_A = 10$ are defined**.**

The shape of the basis vectors of direct and inverse matrix LSP-HS transformations is shown on fig. 3 and fig. 4.

Formula (14) to estimate the spectral envelope of the synthesized speech signal in LSP-HS space is obtained.

8. Conclusions

The method of LSP-HS can be used in matrix form. The matrix form of LSP-HS method makes it possible to employ standard mathematical tools to describe the shape of the spectral envelope of speech signal not only in LPC space, but in the LSP-HS space, which can not be done in the space of classical (first stage splitting) LSP. The matrix form of LSP-HS method allows drawing the alternative structure of analysis filter of speech synthesis, parameters of which are not classic LPC, but the coefficients of LSP-HS space, which reveals the physical meaning of the transition from LPC to the LSP in speech coding algorithms.

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