Image thresholding by cumulative histograms of real and hypothetical images

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Abstract

The new algorithm for image threshold segmentation by cumulative histograms of real and hypothetical images is considered. The algorithm operates with a difference from cumulative histogram of real and hypothetical images. To find the threshold the optimization problems are formulated and solved. Testing and experimental results are presented.

1. Introduction

Indexing is an important tool in CBIR (content-based image retrieval) systems. The formation speed and the adequacy of image features are the major criteria for the identification of the quality of these features used by specified systems.

Determining the image features for indexing requires fast algorithms of image segmentation. Nowadays, there is a great variety of publications on the methods of image segmentation. They can be generally divided into two classes: those that are based on finding the intensity threshold and those that divide the image into regions with certain features. The first ones determine the intensity thresholds based on histograms. Among them, the algorithms of determining the minimal intensity [1], convexity [2], moments [3], entropy [4], minimal errors [5,6] etc. can be distinguished. The typical example of the methods from the second class is the graph-based image segmentation [7]. The drawbacks of the abovementioned and some other algorithms are the different thresholds for similar images even within the algorithms of the same class. Most algorithms are fairly bulky, especially those using graph models or those based on statistical calculations. Modern CBIR-systems process millions of images in real time and therefore need extremely fast and quite accurate image feature determination tools. Segmentation algorithms are an important part of these tools.

In this article, the algorithm from the first class that meets the requirements of automatic CBIR-systems has been presented, namely the one that is simple to develop, has a linear algorithmic complexity and clear physical grounds. We use a cumulative image histogram instead a simple one.

For the simple histogram:

$$V = \sum_{i=1}^{n} V(i) \qquad (1)$$

and for the cumulative histogram:

$$V_F(s) = \sum_{i=1}^{s} V(i) \qquad (2)$$

V is the overall number of image pixels, $V(i)$ is the intensity frequencies, $V_F(s)$ is the accumulating frequency for the given intensity, n is the number of cumulative histogram intervals, s, i are the interval numbers (intensity value).

2. Image intensity segmentation

The task of image segmentation aims at different goals: 1) subtracting the light-gray background from faces and other images; 2) subtracting the black background from the images; 3) dividing the image intensity into two or more parts to process the image part by part; 4) selecting image regions, etc. To find the segmentation threshold the following algorithm is applied.

We use a concept of a hypothetical image, a set of pixels, in which all the intensities are represented by the same number of pixels. The number of pixels for each intensity value is $N \times M / n$, where N, M are the size of the image for which the threshold segmentation is being searched, n is the number of cumulative histogram intervals. For a hypothetical image, a normalized cumulative histogram is constructed according to the following formula:

$$V_{FG}(s) = (1/n) \times s, \quad s = \overline{1,n}, \qquad (3)$$

$V_{FG}(s)$ is the number of pixels (accumulated frequency) of the hypothetical image within the intensity interval 1÷ s.

Let us construct a function of the difference between the cumulative histograms of real and hypothetical images:

$$D(s) = V_F(s) - V_{FG}(s), \quad s = \overline{1,n}. \qquad (4)$$

In Fig.1b the charts of cumulative histograms are presented: the line is the dependence of the hypothetical image, the curve represents the dependence of the "photographer" image (Fig.1a). In Fig.2 the chart of the $D(s)$ function, the difference between the cumulative histograms of the real and hypothetical images, is shown.

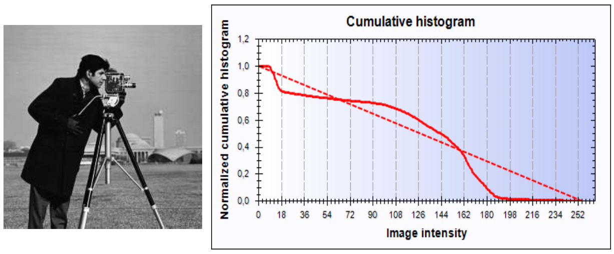


Figure 1: Test image and cumulative histograms.

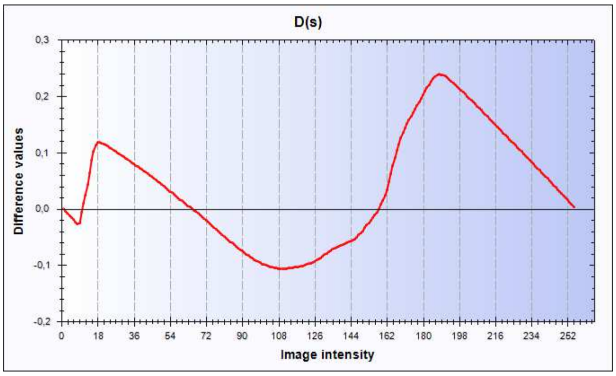


Figure 2: The difference between cumulative histograms.

The *D(s)* function indicates the intervals in which the frequencies of image pixels are larger or smaller than the corresponding values of the hypothetical image when they are increasing or decreasing. The function is characterized by the special points: extrema, inflection points or fracture. In particular, in fig.1c we can see that the *D(s)* function has four extrema: two maxima and two minima. Based on the segmentation tasks, the researcher is interested in the coordinates of the function extreme values from which the increasing or decreasing of the *D(s)* function, and thus the increasing or decreasing of the speed of frequency growth, starts. We consider that the coordinates of the extrema indicate possible thresholds for the image segmentation.

The maximum of the *D(s)* function is prior to the increase in intensity. At the beginning of the intensity interval, the maximum indicates that the left side of the interval contains little information. For this reason, the first left extremum is ignored by the algorithm (rule 1). The minima of the *D(s)* function are considered the threshold values of the segmentation in the middle of the intensity interval since the part on the right side of the interval that could be cut off is non-informative. The last right minimum is ignored by the algorithm (rule 2).

To determine the segmentation threshold, we formulate a one-dimensional optimization problem which aims at finding the values $s_{k\,opt}$ (*k=1,2….*) of the local extrema with which the module of the $D(s)$ function within search intervals has maximum values, and the conditions of the function extrema are preserved:

$$D_m(s_{k\,opt}) = \max|D(s)|,\ \ s \in S_k,\ \ S_1 \cup S_2 ... \cup S_k ... = \overline{1,n};$$
$$\textit{при умові}\ \ \ \Delta D_m(s_{k\,opt})/\Delta s \le \delta,\ \ \ k = 1,2,... \tag{5}$$

S_k is the local extremum search interval, k is the interval number, δ is the error of the derivative value determination. The number and size of the intervals are preset empirically.

The segmentation algorithm control settings are values of the intensity limits: I_g light-gray and black I_b that form the search intervals of the appropriate threshold values.

$$C_g = \{0 \div I_g\},\ C_b = \{255 \div I_b\} \tag{6}$$

The complexity of the problem of determining the limits I_g and I_b of the search intervals for the thresholds is the same as that of the problem of determining threshold values themselves. In our case, we use $I_g = I_b = I_m$, where I_m is the mean value of the image intensity.

Since the $D(s)$ function can contain many extrema, determining the specific threshold value is formulated as an optimization problem:

$$s_{opt} = s_{k\,opt},\ \ (L(s_{opt})) = \min;\ \ k = 1,2,... \tag{7}$$

$L(s_{opt}) = \min$ defines the extremum selection when the additional criterion takes a minimum value. We use: $L(s_{k\,opt}) = I_g - s_{kopt}$ for the light-gray background segmentation and $L(s_{k\,opt}) = 255 - s_{kopt}$ – for the black background segmentation.

For the light-gray segmentation $x_{k\,opt} \in C_g$ and for the black one $x_{k\,opt} \in C_b$.

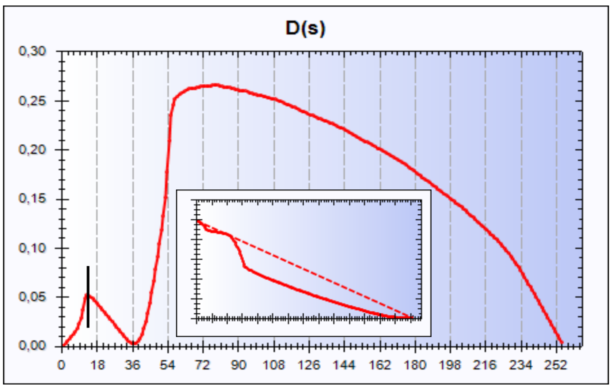
Practically, in problem (7) for the gray background the extremum closest to the I_g value, and for the black one the maximum closest to the black color that is 255, are determined.

For the images with the inflection points on the cumulative histogram charts, the segmentation thresholds are determined by the $D(s)$ function for cutting off the intensity from the opposite sides of the intensity axes.

3. Experimental results

Let us apply the given method to the images taken from [8] (fig.3). The cumulative histograms of the real and hypothetical images as well as the difference function of the cumulative histograms of the real and hypothetical images are shown in fig.4.

Figure 3: Test images from [8].



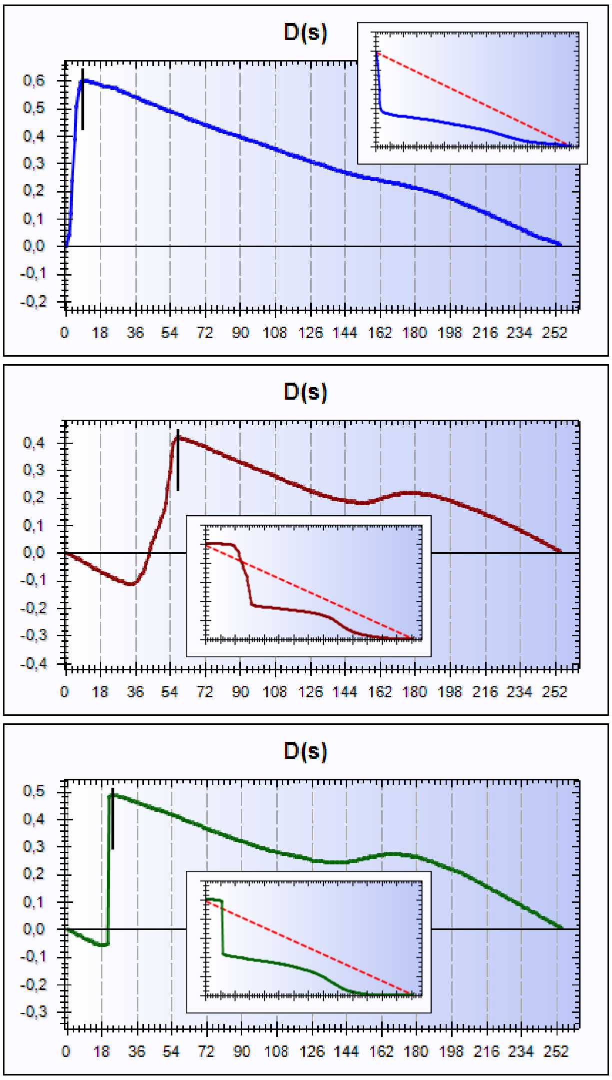


Figure 4: Cumulative histograms of the real and hypothetical images and the difference function of the cumulative histograms the real and hypothetical images for the fig.3.

For the images in fig.4, the determined maxima coordinates of the $D(s)$ function in problem (7) that are closest to the left edge of the intensity interval are used as segmentation thresholds of these images (table 1). The segments, the darkest parts of the images, are presented in fig.5.

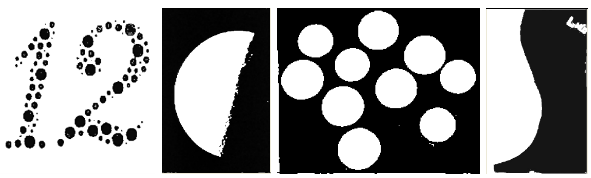


Figure 5: Segmented image parts.

To compare the results of the segmentation with other algorithms, the data from [8] are used. The values of the segmentation thresholds of the given images are shown in table 1, and in fig.6 the segments calculated by means of the algorithms [4,9-11] are presented.

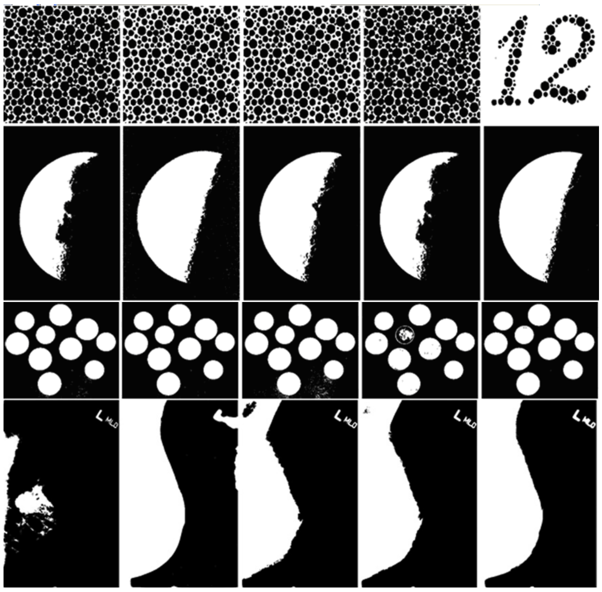


Figure 6: Segmented image parts by means of different algorithms.

The presence in [8] of the histograms that match the histograms of the images copied from the article allow us to compare the thresholds values and the corresponding image segments determined by means of different algorithms.

Table 1: Segmentation thresholds by different methods.

Images	Threshold values					
	Kapur	Rosin	Medina	Otsu	Xu	Cumulative histogram
“Number”	124	82	85	125	47	14
“Moon”	125	13	74	89	33	10
“Coins”	78	79	76	126	88	58
“Mammogram”	185	32	112	99	53	24

The developed method is not sensitive to the size of the segmented images; that is, the size of the “coins” image (220x216) has been reduced and magnified by 1,5 (to 66 and 150 per cent respectively). For the three images different in size, the histogram and the difference function have been determined (fig.7a, 7b). The different size of the same image is proved in the dependence chart of the variance of the pixels coordinates for every intensity value from 0 to 255 of these three images (fig.7c). In light tones the histograms differ a little, but in the cumulative ones vibration smoothing takes place. Thus, this method yields the same thresholds for the images that are different in size if their histograms have the same shape.

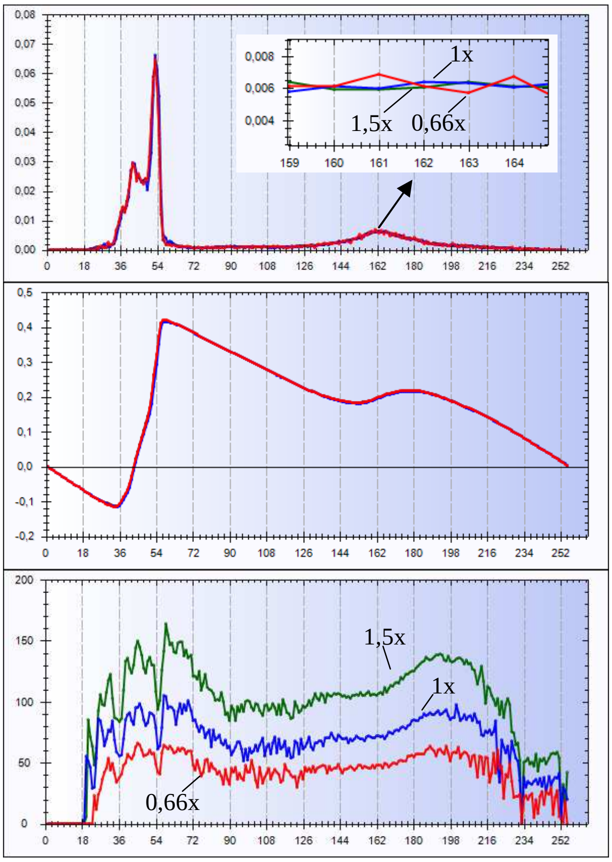


Figure 7: Histograms, differences of the cumulative histograms and variance of the “coins” image pixels coordinates in three different sizes (66, 100, 150 per cent).

In fig.8a the example photograph of a face [12] is shown. The difference function of the cumulative histogram for this photograph and its segmentation are shown in fig.8b,c,d. By the $D(s)$ function and limitation I_g = 85 (mean intensity), a maximum in point 126 has been found, which is used as a segmentation threshold for the gray background. The coordinate of the maximum for the black segmentation equals 27.

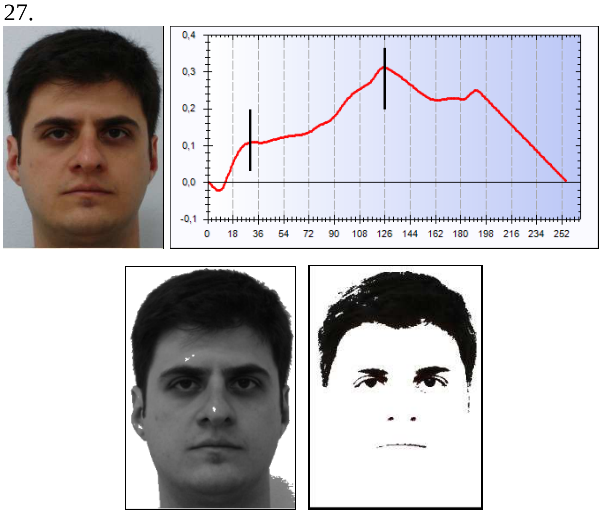


Figure 8: Segmentation of gray and black in the photograph of a face.

4. Conclusion

The method of determining the segmentation thresholds for the image based on the cumulative histogram of real and hypothetical images has been suggested. The process of determining the thresholds is conducted by the search algorithm for the extrema of the one-dimensional difference functions of the cumulative histograms of real and hypothetical images and the proximity of the threshold to search interval edge. The algorithm is characterized by simplicity and the absence of the calculation of any statistical characteristics, the linear algorithmic complexity with respect to size of the image and the intensity interval. It is meant for multiple use in determining image features in CBIR systems.

5. References

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