A mathematical model for generating artificial realistic-form electrocardiogram with internal and external distortions

L.S. Fainzilberg, T.Jy. Bekler

International Research and Training Center of Information Technologies and Systems
Academica Glushkova avenue, 40, Kiev-680, Ukraine, 03680
tanja.bekler@gmail.com

Abstract

A mathematical model for generating of artificial electrocardiograms (ECG) with internal and external distortions is described. Model allows generating different realistic-form ECG with specified amplitude-time characteristics of informative fragments, including generating of T- wave alternans and heart rate turbulence. Simulation results are given.

1. Introduction

Modern development stage of electrocardiography is characterized by wide application of digital electrocardiographs with built-in algorithms for automatic data processing and interpretation of the ECG. At the same time, new nontrivial approaches to signal processing are used to improve the validity and accuracy of diagnostic results.

Construction and quality assessment such algorithms requires using of mathematical models, providing an generating of realistic-form ECG with specified amplitude-time characteristics of informative fragments.

It is known that the real ECG are characterized by complexity and a variety of forms, lack of a clear boundary between separate fragments, changes of a form of the same fragments from a cycle to a cycle. Besides, in actual practice the form of informative fragments is distorted by different internal and external distortions, which can not be reduced only to an additive hindrance. All this complicates constructing problem of adequate models for generating artificial ECG.

In papers [1-3] algorithms based on linear and quadratic interpolation of ECG elements are proposed. However, such models do not allow to generate realistic-form signal and to simulate manifestations of some pathological conditions of cardiovascular system on ECG.

Model [4], in which particular elements are approximated by Gaussian functions, provide more realistic form of "idealized" ECG cycles. At the same time, such algorithm doesn't provide generation of a sequence of cardiac cycles with internal and external distortions. It limits application domain of such models.

In [5] the algorithm for generating of artificial ECG on the basis of operational transformation over separate fragments of "standard" is proposed. And although this approach allows synthesizing sequence of distorted ECG cycles of realistic form, proposed model can not be fully attributed to generative models because the real cycle as reference cycle of ECG is used. In addition, this model does not allow to simulate such an important diagnostic predictor of sudden cardiac death as T-wave alternans [6] because the model does not allow to independently distort of individual fragments of template amplitude due to discontinuity.

One more well-known model is dynamical model for generating synthetic electrocardiogram signals based on computational solution of three coupled ordinary differential equations, which are generate the trajectory in a three-dimensional state space with coordinates (x, y, z) [7]. However, computational solution limits the scope of the model.

A new approach to construction of generative model for creating of artificial ECG is developed at present work. This approach is based on analytical solution of system of the differential equations

$$\dot{x} = \alpha x - \omega y\,, \tag{1}$$

$$\dot{y} = \alpha y + \omega x\,, \tag{2}$$

$$\dot{z} = -\sum_{i \in \{P,Q,R,S,T\}} a_i \Delta\theta_i e^{-\frac{\Delta\theta_i^{\,2}}{2b_i^2}} - (z - z_B)\,. \tag{3}$$

In this system: $\alpha = 1 - \sqrt{x^2 + y^2}$, ω – is the angular velocity of point as it moves in plane (x, y), $\Delta\theta_i = (\theta - \theta_i) \bmod 2\pi$, $i \in \{P, Q, R, S, T\}$, where $\theta \in [-\pi, \pi]$ – is the current angle.

Cyclicity of ECG is simulated by moving of point in (x, y)- plane along variable-length trajectory. Informative fragments of every cycle are modeled by movement of this point in z-direction (fig. 1).

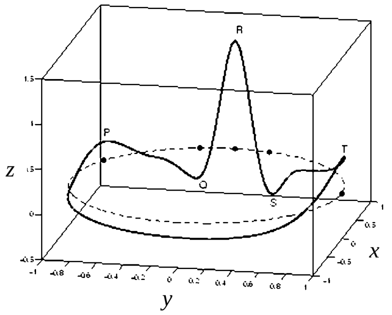


Figure 1: trajectory generated by model (1)–(3)

Time moments t, when P, Q, R, S, T -waves appears, determine the location of limit cycle points that correspond to fixed angles θ_P, θ_Q, θ_R, θ_S, θ_T.

2. Mathematical models of artificial ECG

Generative model of artificial ECG based on the analytical solution of (1) - (3) with techniques-assisted described in [4] is developed.

The sequence $Z_1(t), \ldots, Z_N(t)$ of cycles of artificial ECG is formed according to the template $z(t)$, described as a sum of asymmetric Gaussian functions [8]:

$$z(t) = \sum_{i \in \{P,Q,R,S,ST,T\}} A_i \cdot \exp[-\frac{(t - \mu_i)^2}{2[b_i(t)]^2}]\,, \tag{4}$$

under limits

$$\begin{aligned} 0 \le t_P^{(1)} < t_P^{(2)} \le t_Q^{(1)} < t_Q^{(2)} = t_R^{(1)} < t_R^{(2)} = \\ = t_S^{(1)} < t_S^{(2)} = t_{ST}^{(1)} \le t_{ST}^{(2)} \le t_t^{(1)} < t_t^{(2)} \le t_0 \end{aligned},$$

where t_0 – general cycle time (ms) $z(t)$, connected with heart rate F_{HR} (beat/min)

$$t_0 = \frac{60 \cdot 1000}{F_{\mathrm{HR}}},$$

while the beginnings $t_i^{(1)}$ and endings $t_i^{(2)}$ of every i-th template fragment, $i \in \{P, Q, R, S, ST, T\}$, are connected with parameters $b_i^{(1)}$, $b_i^{(2)}$ and μ_i such way:

$$t_i^{(1)} = \mu_i - 3b_i^{(1)},$$
$$t_i^{(2)} = \mu_i + 3b_i^{(2)}.$$

Parameters A_i and μ_i determine desired levels of amplitudes and time moments, when i-th informative template fragment, $i \in \{P, Q, R, S, ST, T\}$ is taking the maximum value under $A_i > 0$ or minimum value under $A_i < 0$. Parameters

$$b_i(t) = \begin{cases} b_i^{(1)} & \forall\ t \le \mu_i, \\ b_i^{(2)} & \forall\ t > \mu_i, \end{cases}$$

under $b_i^{(1)} \ne b_i^{(2)}$ allow to generate asymmetric fragments including asymmetric T-wave, if $b_i^{(1)} \ne b_i^{(2)}$.

On basis of template (4) on consecutive time-intervals $t_{0m} = t_0(1 + \gamma_m)$, $m = 1, ..., N$ distorted cycles $Z_1(t), ..., Z_N(t)$ are generated by the formula

$$Z_m(t) = \begin{cases} \sum\limits_{i \in \{Q,R,S,ST,T\}} \tilde{A}_{im} \exp[-\frac{(t - \tilde{\mu}_{im})^2}{2\tilde{b}_{im}^2}] + h(t), \\ \qquad \forall\ m \in \{1, \dots, N\} / (I_E \bigcup I_F), \\ Z_E(t) + h(t) \qquad \forall m \in I_E, \\ Z_F(t) + h(t) \qquad \forall m \in I_F, \end{cases} \quad (5)$$

where

$$\tilde{A}_{im} = A_i(1 + \alpha_{im}), \quad (6)$$

$$\tilde{\mu}_{im} = \mu_i(1 + \delta_{im}), \quad (7)$$

$$\tilde{b}_{im} = \begin{cases} b_i^{(1)}(1 + \varepsilon_{im}^{(1)}) & \forall\ t \le \mu_{im}, \\ b_i^{(2)}(1 + \varepsilon_{im}^{(2)}) & \forall\ t > \mu_{im}. \end{cases} \quad (8)$$

Internal perturbations are modeled by distorting of heart rate (time interval t_0) and template parameters A_i, μ_i, $b_i^{(1)}$, $b_i^{(2)}$ on every m-th cycle. Those distortions are based on using of realizations of independent random quantities γ_m, α_{im}, δ_{im}, $\varepsilon_{im}^{(1)}$, $\varepsilon_{im}^{(2)}$, distributed on bounded intervals

$$\gamma_m \in [-\gamma_0, \gamma_0],\ \alpha_{im} \in [-\alpha_i^0, \alpha_i^0],\ \delta_{im} \in [\delta_i^0, -\delta_i^0],$$
$$\varepsilon_{im}^{(1)} \in [-\varepsilon_i^0, \varepsilon_i^0],\ \varepsilon_{im}^{(2)} \in [-\varepsilon_i^0, \varepsilon_i^0]$$

with zero expectations. Here γ_0, α_i^0, δ_i^0, ε_i^0 – constants that define required distortion boundaries.

External perturbations are modeled by additive function

$$h(t) = h_G(t) + h_R(t) + h_D(t)$$

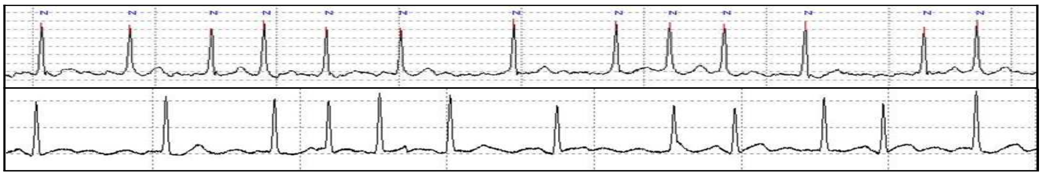
that imitates network noise $h_G(t)$ with given frequencies and amplitudes, muscular tremor in the form of random disturbance $h_R(t)$ with given partition law, and baseline wander as low-frequency function $h_D(t)$ with desired level.

Subsets $I_E \subset \{1, ..., N\}$ and $I_F \subset \{1, ..., N\}$, $I_E \bigcap I_F = \varnothing$, which consist of E and F ordered indexes, determine random points of time, when function $Z_E(t) \in \mathrm{Z}_E$ that simulates extrasystole and function $Z_F(t) \in \mathrm{Z}_F$ that simulates artifact are generated on artificial ECG.

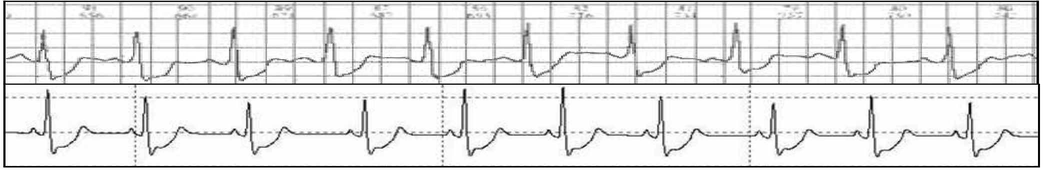
Model (5)–(8) allows to generate realistic-form ECG, including modelling signals with shift of ST-segment above (elevation) and below (depression) baseline, inversion of T-wave, arrhythmias and another electrocardiographic signs of cardiac pathologies.

On fig. 2 simulated results that confirm of adequacy of proposed model are shown.

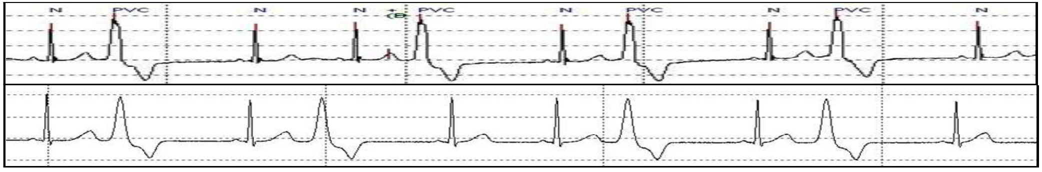
The real ECG with cardiac fibrillation from MIT–BIH Arrhythmia Database (at the top), artificial ECG (at the bottom) are presented on fig. 2, a). The real ECG with ST-segment depression (in compliance with [9]) (at the top), artificial ECG (at the bottom) are shown on fig. 2, b). The ECG with extrasystoles from MIT–BIH Arrhythmia Database (at the top) and artificial ECG (at the bottom) are shown on fig. 2, c).



(a) – Simulation of atrial fibrillation



(в) – Simulation of ST-segment depression



(c) – Simulation of ECG with extrasystoles

Figure 2: Real and artificial ECG

3. Modelling of T-wave alternans and heart rate turbulence

An important characteristic of computer systems for medical diagnostics is the analysis of predictors of sudden cardiac death. One of these predictors is the T -wave alternans which is reduced to alternation of waves with different amplitude, duration, or symmetry.

We introduce some basic hypotheses [10] to construct a model of an artificial ECG with T -wave alternans.

Hypothesis 1. If alternans is absent, the T -wave amplitude is a realization of *one* random quantity Y, which varies relative to some mean value $\mathrm{M}\{Y\}$ with bounded variance.

Hypothesis 2. If alternans is present, T -waves amplitudes are realizations of *two* random quantities Y_A and Y_B, which varies relative to their "own" mean values $\mathrm{M}\{Y_A\} \neq \mathrm{M}\{Y_B\}$ under bounded variance, and

$$Y_A \in [Y_A^{\min}, Y_A^{\max}],\ \ Y_B \in [Y_B^{\min}, Y_B^{\max}] \ \text{ and }$$

$$Y_A^{\max} < Y_B^{\min} \ \text{ or } \ Y_B^{\max} < Y_A^{\min}.$$

Hypothesis 3. If alternans is present, the order of rotation of two wave types can be changed at random time moments $t_1, t_2, ..., t_M$. However, number of such moments M is much less than number of analyzed cycles, and

$$\left|t_i - t_j\right| > T_0 \ \ \forall i, j \in \{1,...,M\},\ i \neq j,$$

where T_0 – minimum time interval that characterizes alternans.

To generate artificial ECG with T -wave alternans we propose to pass from (10) to following model

$$Z_m(t) = \begin{cases} \sum\limits_{i \in \{Q,R,S,ST,T\}} \tilde{A}_{im} \exp[-\dfrac{(t-\tilde{\mu}_{im})^2}{2\tilde{b}_{im}^2}] + h(t), \\ \qquad \forall\, m \in \{1, \ldots, N\}/(I_E \bigcup I_F), \\ Z_E(t) + h(t) \qquad \forall m \in I_E, \\ Z_F(t) + h(t) \qquad \forall m \in I_F, \end{cases} \tag{9}$$

where

$$\tilde{A}_{im} = \begin{cases} A_i(1+\alpha_{im}) & \forall\ i \in \{P,Q,R,S,ST\}, \\ A_T \lambda_m^{(A)} (1+\alpha_{Tm}) & under\ \ i = T, \end{cases} \tag{10}$$

$$\tilde{\mu}_{im} = \mu_i (1+\delta_{im}), \tag{11}$$

$$\tilde{b}_{im} = \begin{cases} b_i^{(1)}(1+\varepsilon_{im}^{(1)})\ \ \forall\ \ t \le \mu_{im},\ \ \forall i \in \{P,Q,R,S,ST\}, \\ b_i^{(2)}(1+\varepsilon_{im}^{(2)})\ \ \forall\ t > \mu_{im},\ \ \forall i \in \{P,Q,R,S,ST\}, \\ b_T^{(1)} \lambda_m^{(b1)} (1+\varepsilon_{Tm}^{(1)})\ \forall\ \ t \le \mu_{Tm},\ \ \ i = T, \\ b_T^{(2)} \lambda_m^{(b2)} (1+\varepsilon_{Tm}^{(2)})\ \forall\ t > \mu_{Tm},\ \ \ i = T. \end{cases} \tag{12}$$

Parameters $\lambda_m^{(A)}$, $\lambda_m^{(b1)}$, $\lambda_m^{(b2)}$ that characterize prescribed levels of alternans of amplitude Δ_A, duration Δ_τ, and symmetry Δ_β are defined by:

$$\lambda_m^{(A)} = \begin{cases} 1 + \dfrac{\Delta_A}{A_T},\ if\ \ \lambda_{m-1}^{(A)} = 1\ \ \forall\ m \in \{1,..,N\}/I_M, \\ 1,\ \text{åñëè}\ \ \lambda_{m-1}^{(A)} = 1 + \dfrac{\Delta_A}{A_T}\ \ \forall\ m \in \{1,..,N\}/I_M, \\ \lambda_{m-1}^{(A)}, \qquad\qquad \forall\ m \in I_M. \end{cases} \tag{13}$$

$$\lambda_m^{(b1)} = \begin{cases} 1 + \dfrac{\Delta_\beta}{b_T^{(1)}} + \dfrac{\Delta_\tau}{6b_T^{(1)}},\ if\ \ \lambda_{m-1}^{(b1)} = 1\ \forall\ m \in \{1,..,N\}/I_M, \\ 1,\ \text{åñëè}\ \lambda_{m-1}^{(b1)} = 1 + \dfrac{\Delta_\beta}{b_T^{(1)}} + \dfrac{\Delta_\tau}{6b_T^{(1)}}\ \ \forall\ m \in \{1,..,N\}/I_M, \\ \lambda_{m-1}^{(b1)}, \qquad\qquad \forall\ m \in I_M, \end{cases} \tag{14}$$

$$\lambda_m^{(b2)} = \begin{cases} 1 - \dfrac{\Delta_\beta}{b_T^{(2)}} + \dfrac{\Delta_\tau}{6b_T^{(2)}},\ if\ \lambda_{m-1}^{(b2)} = 1\ \forall\ m \in \{1,..,N\}/I_M, \\ 1,\ \text{åñëè}\ \lambda_{m-1}^{(b2)} = 1 - \dfrac{\Delta_\beta}{b_T^{(2)}} + \dfrac{\Delta_\tau}{6b_T^{(2)}}\ \ \forall\ m \in \{1,..,N\}/I_M, \\ \lambda_{m-1}^{(b2)}, \qquad\qquad \forall\ m \in I_M. \end{cases} \tag{15}$$

Subset $I_M \subset \{1,...,N\}$ consists of M ordered indexes ($I_M \bigcap (I_E \cup I_F) = \varnothing$) and determines random time moments, when sequence of T -waves on artificial ECG is changed under alternans. Simulation of these effects is necessary when testing of computer analysis algorithms of T -wave alternans.

The real-ECG recording with T -wave alternans (at the top) [11] and artificial ECG (at the bottom) are presented on fig. 3.

Figure 3: Simulation of T -wave alternance

Another important predictor of sudden cardiac death is a phenomenon of heart rate turbulence. It is a short-time fluctuations of sinus cycle durations (RR-intervals), which follows after ventricular ectopic beat [12].

A table model for the generation of ECG with the given values of parameters - TO (turbulence onset) and TS (turbulence slope) is proposed in this work.

Parameter TO is calculated (in %) by formula

$$TO = \frac{(RR_1 + RR_2) - (RR_{-2} + RR_{-3})}{(RR_{-2} + RR_{-3})} \times 100,$$

where RR_{-2}, RR_{-3} – intervals that follow before ventricular ectopic beat, RR_1, RR_2 – intervals, which follow after ventricular ectopic beat.

Parameter TS is calculated by formula

$$TS = \max_j a_j,\ \ j = 1,...,16.$$

a_j – slope coefficients of straight lines, which constructed for every five consecutive RR-intervals of twenty, which follow after ventricular ectopic beat:

$$RR_1 \ldots RR_5,\ \ RR_2 \ldots RR_6,\ \ldots,\ \ RR_{16} \ldots RR_{20}.$$

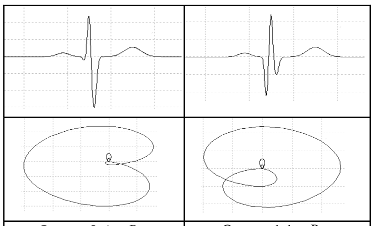
The real-ECG recording with turbulence parameters $TO = -10\%;$ $TS = 2{,}6$ ms/RR is presented on fig. 4 at the top. Rhythmogram that corresponding to ECG is shown at the bottom of the figure.

Figure 4: Simulation of heart rate turbulence

4. Research of new signal processing technique

One of new signal processing techniques, which was confirmed in clinical conditions based on an analysis of ECG in phase space. A number of properties established connection of diagnostic features that characterize shape of phase portrait of ECG with parameters of the model that generates ECG in time domain are discovered [13].

It is established that phase portrait orientation angle W is determined by ratio of amplitudes of Q and S waves (fig. 5). Statistical dependence between these parameters can be described by regression equation $W = 200{,}85e^{-0{,}7928\rho}$, where $\rho = Q/S$. Determination coefficient is $R^2 = 0{,}986$.



$Q = -0{,}1$ *мВ* $S = -1{,}5$ *мВ* $W = 186\ °$	$Q = -1{,}1$ *мВ* $S = -0{,}5$ *мВ* $W = 34{,}6\ °$

Figure 5: ECG cycles in time domain and phase coordinates

Also was shown that the diagnostic character β_T of symmetry of repolarization fragment is connected with parameters $b_T^{(1)}$ and $b_T^{(2)}$ of proposed model. Dependence between these parameters can be described by function $\beta_T = 1{,}0082\eta^{-0.4248}$, where $\eta = b_T^{(1)} / b_T^{(2)}$. Determination coefficient is $R^2 = 0{,}993$.

It is established also that "dispersion" parameter D of phase trajectories is connected with limitation ε_0 that applied to distortions of R-wave amplitudes of model. Dependence between these parameters can be described by linear regression equation $D = 47{,}841\varepsilon_0 - 1{,}3172$, correlation coefficient $r = 0{,}997$.

5. Conclusions

As seen from fig. 2 and 3, forms of artificial ECG are close enough to forms of real signals. An advantage of artificial signals is that user exactly knows its amplitude-time characteristics, which expand application scope of these signals.

On the basis of described models program-technical complex was built. It is a handy tool for quality testing of new algorithms for analysis and interpretation of ECG. It is also used for testing of characteristics and consumer qualities of digital electrocardiographs, in particular, device FASEGRAF® in its production and during service [14].

Also statistical dependencies between diagnostic features of ECG in time domain and phase space coordinates are constructed for the first time.

Proposed mathematical models for generation of artificial ECG can also be used in other areas, in particular, for scientific researches or as tutorial for medical students which study electrocardiography.

References:

[1] Вайсман М.В., Прилуцкий Д.А., Селищев С.В. Алгоритм синтеза имитационных электрокардиосигналов для испытания цифровых электрокардиографов // Электроника. – 2000. – № 4. – С. 21-24.

[2] Ладяев Д.А., Федосин С.А. Моделирование ЭКГ-сигнала // Информационные технологии моделирования и управления. –2006. – № 6 (31). – С. 702-709.

[3] Белоцерковский О.М., Виноградов А.В., Галатян Э.Э., Тарасов А.С., Шебко С.В. Способ кодирования данных ЭКГ в модели контурного и динамического анализа ЭКГ // Компьютер и мозг. Новые технологии. – М.: Наука, – 2005. – С. 241-255.

[4] Абрамов М.В. Аппроксимации экспонентами временного кардиологического ряда на основе ЭКГ // Вестник кибернетики. – Тюмень: ИПОС СО РАН, – 2010. – № 9. – С. 85-91.

[5] Файнзильберг Л.С. Компьютерный анализ и интерпретация электрокардиограмм в фазовом пространстве // Системні дослідження та інформаційні технології. – 2004. – № 1.– С. 32-46.

[6] Predescu D., Mitrut P., Guica A. Microvolt T wave alternans (MTWA) - a new non-invasive predictor of sudden cardiac death. // Romanian Journal of Internal Medicine. – 2004. – Vol. 42. – No. 3. – P. 647-656.

[7] McSharry P.E., Clifford G., Tarassenko L., Smith L.A. A dynamical model for generating synthetic electrocardiogram signals // IEEE Transaction On biomedical Engineering. – 2003. – No. 3. – P. 289-294.

[8] Беклер Т.Ю., Файнзильберг Л.С. Информационная технология тестирования цифровых электрокардиографов с встроенными алгоритмами обработки данных // Управляющие системы и машины. – 2012. – №5(в печати).

[9] Лутай М.И., Голикова И.П., Немчина Е.А. Стабильная стенокардия напряжения и методы ее диагностики // «Здоровье Украины». – 2008. – № 11-1. – С. 18-21.

[10] Файнзильберг Л.С., Беклер Т.Ю. Моделирование альтернации зубца Т на искусственной электрокардиограмме в условиях внутренних и внешних возмущений // Международный научно-технический журнал «Проблемы управления и информатики». – 2012. – № 4. С. 116-128.

[11] Интернет ресурс: http://research.vet.upenn.edu/smallanimalcardiology/ECGTutorial/AbnormalECGs/tabid/4960/Default.aspx

[12] Цветникова А.А. и др. Турбулентность сердечного ритма: методические аспекты. – С.-Пб.:Инкарт, 2008.–32с.

[13] Файнзильберг Л.С., Беклер Т.Ю., Глушаускене Г.А. Математическая модель порождения искусственной электрокардиограммы с заданными амплитудно-временными характеристиками информативных фрагментов // Международный научно-технический журнал «Проблемы управления и информатики». – 2011. – № 5. – С. 61-72.

[14] Файнзільберг Л.С., Беклер Т.Ю. Програмно-технічний комплекс для генерації штучних електрокардіограм реалістичної форми // Науково–практичний журнал «Наука та інновації». – 2012. – Т.8. – № 4. – С. 89-98.