Nonquadratic Regularization for Edge-Preserving Piecewise-Constant Image Restoration

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Abstract

Problem of piecewise-constant image restoration is considered in the paper. Algorithm based on solution of nonlinear nonquadratic regularization problem is developed and discussed in the paper. Nonenergy measure of signal in the form of time duration is modified in order to describe spatial extension of image and used afterwards as a nonquadratic regularization term in image restoration procedure. Proposed approach is investigated through numerical simulations for one-dimensional signals and images, and compared to results given by standard Tikhonov regularization scheme. Results of application of developed algorithm for filtration of noisy aerospace images are given.

1. Introduction

Image filtration problem is rather general and frequently arises in various practical signal and image applications. In this paper we consider the noise suppression problem for class of piecewise-constant images, which applies to several classes of real aerospace images. Overall filtration techniques can be divided into two different classes: approaches using local filtration and global filtration methods aimed at restoration of signal. Latter are usually discussed in wider framework of inverse problem of signal or image deconvolution in noisy environment. Many methods, based either on local or global filtration, were developed. Local image filtration approaches include traditional nonlinear filtration techniques using various operators and masks [1], as well as recently developed ones[2,3], oriented on suppression of different types of noise. Among these encouraging robust results are demonstrated by myriad filtration approach [4]. Alternative way is solution of deconvolution problem with some a priori information about either convolving operator or desired solution. In presence of noise considered problem is ill-posed and usage of some regularization techniques is required. Traditional regularization approach is connected with usage of standard Tikhonov [5] regularization scheme. Main problem here is that, though introducing an effective tool for additive noise cancellation, appropriate additive term additionally blurs processed signal or image. Number of methods were developed using special approximation error cost functions as nonquadratic regularization terms, significantly improving results of image restoration, allowing to take into account piecewise-constant nature of image, detailed comparison of computational effectiveness of these techniques is given in [6]. Yet, majority of proposed forms of cost functions are empirical and their origin can not be justified clearly enough.
In this paper use of information possibilities of the method of minimum of duration (MMD) [7-9] is proposed for efficient solution of image restoration problem. In two-dimensional case considered method is called the method of minimum of extension (MME). Starting from classical Tikhonov regularization technique, modification of regularization term is considered in overall functional to be minimized, taking into account fact of finiteness of either image spatial extent or first derivative of image spatial extent.
Paper is organized as follows. In the second section some brief information about MMD/MME is given and traditional regularization technique after Tikhonov is discussed and proposed regularization method is described. In the third section proposed approach is investigated through numerical simulations for one-dimensional signal and sample synthesized image. In the fourth section application of proposed method for real aerospace image is considered.

2. Nonquadratic regularization approach

Usage of MME for piecewise-constant signals and images is motivated by the fact, that these signals and images and its derivatives can be referred to the class of finite signals.

2.1. MMD/MME foundations

MMD was originally introduced for the problems of finite spectrum signal restoration [7]. Its main idea is use of concept "duration" as non-energy measure of signals. For the purpose of time length description of signal $r(t)$ special discontinuous function χ is introduced in the following form: $\chi[r(t)]=1$ if $r(t)\neq 0$ else $\chi[r(t)]=0$. Result of application of such function gives the so-called "strict duration" of considered signal, measure of its non-zero values, thus giving possibility to formalize the term "finite signal". But use of χ causes difficulties, as in presence of noise with arbitrarily small dispersion the function χ, describing signal under consideration, will never have zero values. Besides, function χ is not suitable for optimization purposes because of its discontinuity. These difficulties have been resolved by use of the approximation of function χ for practical calculations by special function in the following form $\psi[r(t)]=[|r(t)/\lambda|^2+\alpha^2]^{\beta}-\alpha^{2\beta}$; $0<\lambda<\infty$, $\beta=1/n$, $n>2$, with parameters α , β to control its behavior, parameter λ is used for amplitude normalization of processed signal. Result of integrating the proposed function ψ over observation time interval $[-T/2;T/2]$ gives the so-called "generalized duration" or "quasiduration" of considered signal $r(t)$:

$$D_{\alpha,\beta} = \int_{-T/2}^{T/2} \{[|r(t)/\lambda|^2 + \alpha^2]^{\beta} - \alpha^{2\beta}\} dt\ , \qquad (1)$$

where α , β – parameters of functional $D_{\alpha,\beta}$, that can be chosen as [8]: $\alpha = \sigma / \lambda$, σ^2 – additive noise dispersion; β =1/16; $\lambda = \max\{|r(t)|\}$. It is necessary to minimize functional $D_{\alpha,\beta}$ by varying signal $r(t)$. Application of introduced function ψ allows overcoming mentioned above problems. Numerical simulations have shown that quasiduration value in general does not coincide with true duration of signal, but has the same dynamics. Minimization of quasiduration gives approximately the same result as minimization of true duration, but results are more robust.

2.2. Traditional Tikhonov regularization

Setting of image filtration problem usually does not require solution of inverse problem in general, thus no a priori knowledge is needed about apparatus function of system forming the image or system distorting the image (in particular, when point spread function is "quite good"). Application of Tikhonov scheme for construction of regularizing algorithm for solution of incorrect inverse problems provides convergence of regularization algorithm, and turns incorrect instable problem into correct one, suppressing additive noise, which distorts the image. In considered situation for image filtration purposes one can obtain "smooth" approximation of true image on the basis of noisy image, and this approximation will fit the noisy image optimally in the least squares sense.
Filtration technique, based on Tikhonov regularization algorithm, in one dimensional case can be described in following terms. Consider for measured data of some unknown function $\bar{u} \in W_2^1$ following approximation $u_\delta = \|\bar{u} - u_\delta\|_{L_2} \le \delta$ is given. Such function $\tilde{u} \in W_2^1$ must be estimated, that $\|\tilde{u} - \bar{u}\|_{W_2^1} \to 0, \quad \delta \to 0$. This problem is solved by unconstrained minimization of Tikhonov functional of this kind:

$$E_\alpha(\tilde{u}) = \|\tilde{u} - u_\delta\|_{L_2}^2 + \gamma \left\| \frac{d}{dx}\tilde{u} \right\|_{L_2}^2 , \qquad (2)$$

with γ as a regularization parameter. The first term here is the approximation error between estimated and measured functions, and the second one is a regularization term. Afterwards functional (2) minimization problem can be reduced to Euler equations system solution. As an alternative, numerical optimization methods can be applied for direct minimization, which can be of help when Euler equation can not be solved analytically or solution is hard to be obtained. Regularization parameter γ is chosen by discrepancy scheme. An important property of such approach is a priori information about presence of estimated function in Sobolev space, i.e. taking into account smoothness of function. If approximated function can not considered as smooth, or, in particular, is strictly finite, considered approach can not be applied because of intensive blurring of image under processing. Thus special techniques must be used exploiting the a priori knowledge about finiteness of estimated function or first derivative of estimated function, allowing to take into account piecewise constant nature of signal or image.

2.3. Proposed Regularization by MME

According to [9], instead of use of quadratic regularization, based on energetic term, use of nonenergy regularization term is proposed, taking into account time duration or spatial extent of estimated solution.
Then problem in one-dimensional case is formulated as following. Such function $\tilde{u}$ must be found, that condition $\|\tilde{u} - u_\delta\|_{L_2} \le \delta$ is fulfilled, and estimated function has finite time duration or spatial extent. Unconstrained problem formulation is given by functional of such kind:

$$D_{\alpha,\beta}(\tilde{u}) = \|\tilde{u} - u_\delta\|_{L_2}^2 + \gamma D_{\alpha,\beta}(\frac{d}{dx}\tilde{u}) \qquad (3)$$

to be minimized, with γ as regularization parameter. Functional $D_{\alpha,\beta}$ introduces quasiduration of solution derivative as nonquadratic regularization term. It must be noted, that for two-dimensional case functional (3) has sense of "generalized spatial extent" of obtained solution. The solution, which can be obtained by minimization of (3), analogously to [9] is called first derivative extension minimum solution (FDEMS). Previous numerical simulations show that one can take into account intensiveness of present on image additive noise through varying parameter α and β values of functional $D_{\alpha,\beta}$. According to [7-9], if $\alpha^2 >> |\dot{\tilde{u}}|$, then FDEMS tends to solution, which is obtained by standard quadratic regularization method with additive energy term. Otherwise we have a difficult problem of nonlinear minimization to be solved. In [9] it was shown that some optimal value for γ exists so that we would not get neither degenerate zero FDEMS (for very large γ) nor bad FDEMS without regularization (for very small γ).

3. Numerical Simulations

For detailed investigation of possibilities of application of proposed technique for additive noise suppression when filtrating finite objects images and comparison with results of standard quadratic regularization scheme for filtration purposes numerical simulations were made for the cases of synthesized one- and two-dimensional signals: signal, consisting of several finite pulses and finite objects images on some background.

3.1. 1-D signal

Sample one-dimensional signal for processing is represented by superposition of rectangular pulses corrupted by additive Gaussian noise with SNR +15 dB and is shown on Fig. 1. Results of signal restoration given by minimization of functional (2) and (3) for optimal regularization parameter values are shown on Fig. 2 and Fig. 3 appropriately. It is easy to see that while the first signal restored by standard regularization technique is rather blurred, the second one preserves finiteness, thus proposed approach gives very good results with round mean squared error less than 2% between true and restored images.

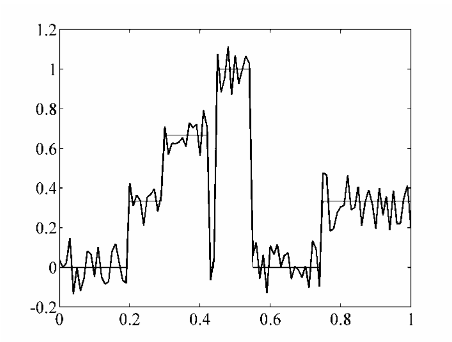

Figure 1: Rectangular pulses corrupted by additive Gaussian noise: initial signal (thin) and noisy signal (thick)

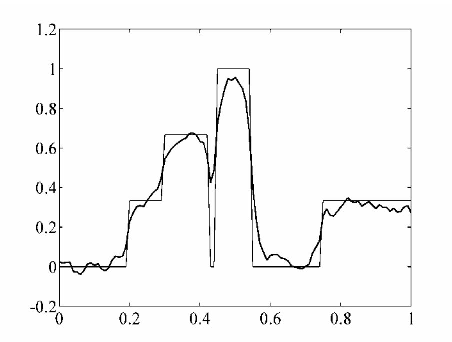

Figure 2: Result of signal restoration by standard regularization technique: estimated signal (thick) and true signal (thin)

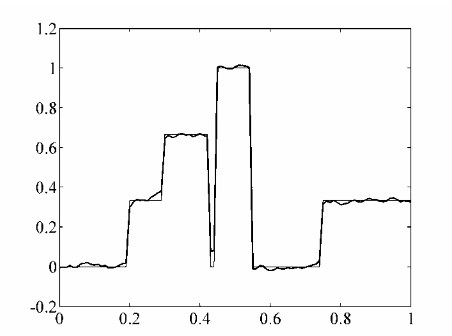

Figure 3: Result of signal restoration using nonquadratic regularization after MMD/MME: estimated signal (thick) and true signal (thin)

3.2. Piecewise-constant image

Consider simulated image of several finite objects located on grey background (Fig. 4) corrupted by intensive additive Gaussian noise with SNR +15 dB (Fig. 5). Results of image restoration given by minimization of functional (2) and (3) for optimal regularization parameter values are shown on Fig. 5 and Fig. 6. Results given by standard regularization have rather poor visual quality, image is very blurred, and in particular thin borders are completely missing, which is unacceptable for detailed analysis. while the image restored through nonquadratic regularization has rather high quality, with almost all informative structural elements undistorted.

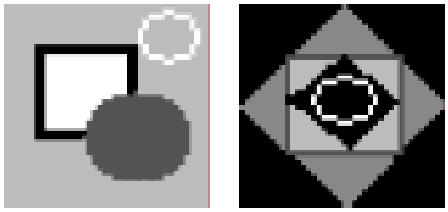

Figure 4: Initial piecewise-constant images

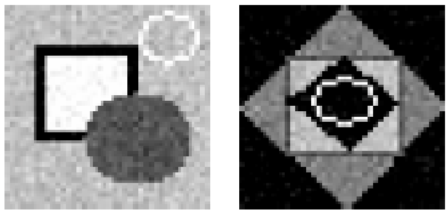

Figure 5: Piecewise-constant image corrupted by additive Gaussian noise

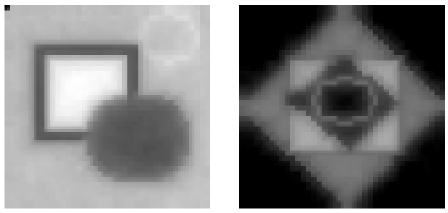

Figure 6: Result of piecewise-constant image restoration by standard regularization technique

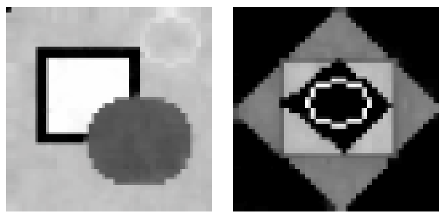

Figure 7: Result of signal restoration using nonquadratic regularization after MMD/MME

4. Experimental Image Processing

As an example of real piecewise-constant image for edge-preserving restoration aerospace image is considered, corrupted by intensive additive noise with SNR +7 dB. Using FDEMS, one can see that only very small details were lost, from comparison of images shown on Fig. 10 and Fig. 8 while blur given by standard regularization technique is unsatisfactory.

Figure 8: Aerospace image corrupted by additive noise

Figure 9: Image restored by standard regularization technique

Figure 10: Image restored using nonquadratic regularization after MMD/MME

5. Conclusions

Proposed nonquadratic regularization method for edge-preserving image filtration shows high performance when processing piecewise-constant images in comparison with classical quadratic regularization approach. Functional of minimum of extension is used in order to introduce nonenergy term in the form of derivative of spatial extent of image or time duration of signal. Overall described approach can be considered as generalization of existing nonlinear nonquadratic techniques, as function taking into account time duration of signal or spatial extent of image can be chosen in various forms.

6. References

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