The Contour Analysis for Hierarchical Recognition of the Gray Scale Images

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Abstract

The patterns formation model of the hierarchical structure objects on the gray scale images in view of the semantic importance of levels and objects is offered. The approach to its realization using wavelet transform is considered. The developed model enables to create flexible models of images processing, to avoid necessity of repeated comparison of the received information with a lot of models during objects classification, to receive the universal approach for the decision of a lot of applied tasks.

1. Introduction

In many practical applications, such as quality surveillance of industrial products, medical diagnostic, ecological monitoring etc., the intellectual images processing and recognition systems (IIPRS) are used, in which the object of recognition has hierarchical structure: "object – subobject – … – elementary object". The basic function of such systems is the extraction of the information essential for functioning of the appropriate application, the kind and form of information representation is defined at a design stage of IIPRS. For the formal description of the hierarchical structure object (or process) a pattern formation model, which correct choice defines efficiency of IIPRS, is used. However, known models take into account, as a rule, only properties of recognition object (structure, the size etc.) [1], which does not satisfy the increased requirements of universality and reliability of IIPRS. Therefore, the article proposes the patterns formation model of the hierarchical structure objects taking into account both features of recognition object, and requirements to the taken information determined by the purpose of processing – its semantic importance.

2. Pattern formation model of the hierarchical structure objects

The hierarchical structure object can be presented as set of the images of a different level of hierarchy (pyramidal representation) [1, 2]:

$$F(x_1, x_2) = \sum_{j=1}^{M} I_j(x_1, x_2) \quad (1)$$

where M – quantity of hierarchy levels on the image; $I_j(x_1, x_2)$ – image on *j*-level of hierarchy (*j*=1, …, *M*).

Turning to the contour description of **IKO** as a set of initial vectors of contour features, to take into account the purpose of information reception it is offered to enter λ_{ji} – parameter of the semantic importance of *i*-object (subobject) on a hierarchy *j*-level (in case only one object is present at the given level this parameter determines the importance of the level)

$$\mathbf{IKO} = \bigcup_{j=1}^{M} \bigcup_{i=1}^{N_j} (\mathbf{K}_{ji})_{\lambda_{ij}} , \quad (2)$$

where $\mathbf{K}_{ji}$ – ordered set of points with coordinates $\{x_{1iq}^{j}, x_{2iq}^{j}\}$ of a some *i*-object contour on a plane at a *j*-level; M – quantity of levels; N – quantity of objects and subobjects; λ_{ji} – importance *i* - object on a *j*-level (from 0 up to *N*).

On sets $\mathbf{K}$ accessory relation is determined, at which one or several subobjects settle down inside other object or subobject. On the basis of model (2) by transformation $\nu(\cdot)$ or composition of transformations $(\ldots(\nu_2(\nu_1(\cdot))))$ it is possible to receive topological features (object quantity, measures of mutual location of objects or subobjects), spectral, geometrical and other features of object. Then the formal description of object with the account (2) can be presented:

$$\mathbf{C}_{\mathbf{IKO}} = \nu_m(...(\nu_2(\nu_1(\mathbf{IKO})))) . \quad (3)$$

Thus, on the basis of the theoretical-plural approach, the patterns formation model of hierarchical structure object looks like:

$$\mathbf{MKO} = \{J, \Lambda, I, \chi, \mathbf{C}_{\mathbf{IKO}}\} , \quad (4)$$

where J – set of hierarchy levels; Λ – set of meanings of the semantic importance of hierarchy levels ; I – set of subobjects; χ - accessory relation, at which one or several subobjects settles down inside other object or subobject.

3. Model realization using wavelet transform

Base procedure allowing going to model (4) is the contour images segmentation, i.e. reception of the contour description of the image (2). In [1, 2] it is shown, that the contour description at different hierarchy levels can be received in the space of wavelet-transformation (WT), for which the frequency-spatial localization is peculiar.

The procedure of hierarchical contour segmentation in WT space is carried out in two stages: detection of edges and getting of the contour description. There are expanding spatial localization of object, the underlining of object edges and their detection on the first stage. The second stage – the ordered file of coordinates of contour points is received.

The continuous wavelet transform of function $f(x) \in L^2(R)$ can be used to underline edges [1]

$$Wf(s, x) = \frac{1}{\sqrt{s}} \int_{-\infty}^{+\infty} f(t) \psi\left(\frac{t-x}{s}\right) dt , \quad (5)$$

where s – scale factor; $\psi\left(\frac{t-x}{s}\right)$ – wavelet, received on the basis of basic function $\psi(t)$ by shift and change of scale; t – variable integration.

For the given task valid wavelets, given as symmetric odd functions $\psi(t)=-\psi(t)$ having the compact or effective carrier can be applied, in particular, Gauss wavelet, hyperbolic wavelet transform etc. [1, 2].

Let's consider representation of edge in space of wavelet-transformation. Taking into account the final sizes of the image, the model of ideal step edge for a row of the image looks like:

$$f(x)=\begin{cases} h_1, & 0<x<x_0 \\ h_2, & x_0 \le x \le x_{\max} \end{cases}, \quad (6)$$

where x_0 – coordinate of step edge point, h_1, h_2 – meaning of a background and signal intensity accordingly, x_{max} – row length. The wavelet transform of step edge model looks like

$$Wf(s,x)=\frac{1}{\sqrt{s}}\int_0^{x_0} h_1\psi\left\{\frac{t-x}{s}\right\}dt+\frac{1}{\sqrt{s}}\int_{x_0}^{x\max} h_2\psi\left\{\frac{t-x}{s}\right\}dt \quad (7)$$

After replacement variable and integration we have

$$Wf(s,x)=-\sqrt{s}\left(h_1F\left(-\frac{x}{s}\right)+(h_1-h_2)F\left(\frac{x_0-x}{s}\right)+h_2F\left(\frac{x_{\max}-x}{s}\right)\right), \quad (8)$$

where $F(x)=\int\psi(x)dx$.

The received expression has three components, each of which is located in a neighborhood of the certain point (0, x_0, $x_{\max}$ accordingly). The first and the last components arise in connection with the boundary effect and are not taken into account at the analysis. The second component defines presence of step edge, which corresponds with extremum. Let's find derivative:

$$\frac{\partial Wf(s,x)}{\partial x}=-\frac{h_1-h_2}{\sqrt{s}}\psi\left(\frac{x_0-x}{s}\right)=0. \quad (9)$$

The equation solving is in a point of crossing of an axis x (for example, for Gauss wavelet $\psi(x)=-xe^{-x^2}$ extremum's coordinate $x^*=x_0$ coincides with coordinate of a point of step edge). Thus, to find a step edge in a row of the image it is possible to implement its continuous wavelet transform and to find local extremum points.

In real images we can often meet a ramp edge, which model can be submitted as

$$f(x)=\begin{cases} h_1, & 0<x<x_0, \\ h_1+\dfrac{h_2-h_1}{d}(x-x_0), & x_0\le x\le x_0+d, \\ h_2, & x_0+d\le x\le x_{\max}, \end{cases} \quad (10)$$

where d – extent of ramp edge.

The similar mathematical calculations for WT of model (10) allow receiving the following equation:

$$\frac{1}{\sqrt{s}}\frac{h_2-h_1}{d}\int_{x_0}^{x_0+d}\psi\left\{\frac{t-x}{s}\right\}dt=0. \quad (11)$$

The function $\psi(t)$ within a designation variable coincides with wavelet; hence, it is odd function. For an odd function the integral in symmetric limits is equal 0. Then the equation (11) has decision

$$x=x_0+\frac{d}{2}. \quad (12)$$

Thus, in a point $x^*=x_0+\frac{d}{2}$ conterminous to the middle of ramp edge, takes place extremum of WT when symmetric odd function is used as wavelet. The extremum coordinate does not depend on scale s of transformation.

Thus, the WT properties allow using it for localization both step edge and ramp edge.

For a considered class of basic functions $\psi(\frac{t-x}{s})$ the wavelet transform of some function $f(x)$ can be replaced by operation of convolution. Really, we shall use definition of wavelet transform (5) and wavelet property of oddness

$$Wf(s,x)=\left\langle f(x),\frac{1}{s}\psi(\frac{x}{s})\right\rangle=\frac{1}{\sqrt{s}}\int_{-\infty}^{+\infty} f(t)\psi\left(\frac{t-x}{s}\right)dt=$$
$$=-\frac{1}{\sqrt{s}}\int_{-\infty}^{+\infty} f(t)\psi\left(\frac{x-t}{s}\right)dt=-(f(x)*\frac{1}{s}\psi(\frac{x}{s})), \quad (13)$$

where $*$ – convolution operation.

Hence, it is possible to use for the analysis both WT properties, and properties of convolution, in particular in frequency space.

4. Objects edge detection under the scheme of the approached method

The edge detection includes three basic procedures [1]: expansion of spatial localization of object boundaries; underlining of edge; detection of object boundaries (for example, at threshold processing).

The transition in WT space allows combining the first two of them and, taking into account, as was shown above, that the detection of object points at transition of wavelet-transformation space is reduced to a finding of extremum coordinates of the transformed function, can be carried out on the basis of the scheme of the approached method [3.]

Let's consider the image as two-dimensional function $F(x_1,x_2)$, whose meaning in a point with spatial coordinates (x_1,x_2) is positive scalar value, the physical sense is defined by the input image. In case of one-dimensional processing on row or on column $f(x)$ the operator A of detection of edge points carries out transformation $\mathrm{A}f(x)\rightarrow KP(x)$, where $KP(x)$ – contour preparation of a row, or column of the input image. Typically, some of the differential edge detection operators are applied as the operator A.

Transition in WT space (operator W_n), which is offered to be used at contour segmentation in this article, has the following advantages: allows to expand spatial localization of object, that provides an opportunity of iterative search methods application; raises noise proof features at allocation of objects boundaries; allows to adjust detailed elaboration at underlining difference of intensity and to take into account the geometrical sizes of objects, providing the hierarchical approach to contour segmentation.

The task of object edge detection at representation of the image in WT space can be considered as a task of one-dimensional $f_n(x)$ or two-dimensional $F_n(x_1, x_2)$ function extremum search in noise conditions. With such approach to edge detection in WT space it is offered to apply the operators A_n realizing an iterative (approached) method [3] that allows to find the decision with the given accuracy and to reduce computing expenses. The accuracy of reception of the contour preparation is checked on the reference images on the basis of difference norm minimization $\|\varphi_n A_n W_n f(x) - Af(x)\| \to 0$. The operator φ_n is a scaling operator.

The basis for usage of iterative search algorithms is the property of WT to achieve extremum in a point of intensity sharp change (i.e. edge point), and also equality to zero of result double WT.

The analysis of known gradient iterative search methods has shown that their drawbacks connected with calculation of a gradient: a low noise stability and slow speed of convergence, when the received meanings of a gradient are small, can also be eliminated by transition in WT space.

Really, let the function $F(x_1, x_2)$ be twice continuously differentiated in δ-vicinities of a stationary point (x_1^*, x_2^*) on an interval of search of extremum, i.e., therefore it is an even function on x_1 and on x_2. Then WT function $F(x_1, x_2)$ with odd basic functions $\psi(x_1)$ and $\psi(x_2)$:

$$WF_1(s, x_1, x_2^*) = \frac{1}{\sqrt{s}} \int_{-\infty}^{+\infty} F(t, x_2^*) \psi\left(\frac{t - x_1}{s}\right) dt,$$

$$WF_2(s, x_1^*, x_2) = \frac{1}{\sqrt{s}} \int_{-\infty}^{+\infty} F(x_1^*, t) \psi\left(\frac{t - x_2}{s}\right) dt \qquad (14)$$

in a stationary point (x_1^*, x_2^*) are equal to zero as integrals from odd function in symmetric limits, i.e. $WF_1(s, x_1^*, x_2^*) = 0$, $WF_2(s, x_1^*, x_2^*) = 0$, or in operator kind

$$WF(X^*) = 0. \qquad (15)$$

Thus to finding coordinates of extremum points it is necessary to solve operator equation (15). To solve this equation it is offered to use a rule of Newton to find the following approximation on previous [3]

$$X_{k+1} = X_k - [\nabla WF(X_k)]^{-1} WF(X_k), \qquad (16)$$

where k – number of iteration, $\nabla WF(X_k)$ – gradient $WF(X)$ in a point X_k. The finding $\nabla WF(X_k)$ can be replaced by calculation WT of $WF(X)$ with odd basic function (not necessarily by what was used for reception $WF(X)$).

The conformity meanings of partial derivative and WT in a point of extremum (x_1^*, x_2^*) of function $F(x_1, x_2)$ are shown in the table 1. It is necessary to notice, that WT and appropriate partial derivative in a neighborhood of extremum have opposite signs. In view of stated the kvazi Newton method search of extremum of one-dimensional function in WT space can be submitted

$$x_{k+1} = x_k + [W\{Wf(x_k)\}]^{-1} Wf(x_k). \qquad (17)$$

It is possible to show, that for definition of intensity difference (6) or (10) by iterative kvazi Newton method in view of WT properties (17) gets a kind

$$x_{k+1} = x_k - [Wf(x_k)]^{-1} W(Wf(x_k)). \qquad (18)$$

Table 1: The properties of partial derivative and WT

Meanings	
partial derivative	WT
$\dfrac{\partial F(x_1^*, x_2^*)}{\partial x_1} = 0$	$W_1 F(x_1^*, x_2^*) = 0$
$\dfrac{\partial F(x_1^*, x_2^*)}{\partial x_2} = 0$	$W_2 F(x_1^*, x_2^*) = 0$
$\dfrac{\partial F(x_1^* - \Delta x, x_2^*)}{\partial x_1} \cdot \times \dfrac{\partial F(x_1^* + \Delta x, x_2^*)}{\partial x_1} < 0$	$W_2 F(x_1^* - \Delta x, x_2^*) \cdot W_2 F(x_1^* + \Delta x, x_2^*) < 0$
$\dfrac{\partial F(x_1^*, x_2^* - \Delta x)}{\partial x_2} \times \dfrac{\partial F(x_1^*, x_2^* + \Delta x)}{\partial x_2} < 0$	$W_1 F(x_1^*, x_2^* - \Delta x) \cdot \cdot W_1 F(x_1^*, x_2^* + \Delta x) < 0$

The algorithm of reception of a contour preparation *KP* for step edge or ramp edge on a basis of iterative kvazi Newton method of extremum search of function (18) is given below.

Step 0. The initialization is made: the value of an error θ is set, number of a line i = 1, $i = \overline{1, i_{\max}}$ is accepted.

Step 1. For accepted number of a row the initial meaning $k = 0$ is set and gets out x_k.

Step 2. WT $Wf_i(x_k)$ of function $f_i(x)$ is determined.

Step 3. Repeated WT of function $f_i(x)$ in a point x_k, i.e. $W(Wf_i(x_k))$ is determined.

Step 4. k+1 approximation x_{k+1} by algorithm (18) is obtained.

Step 5. If $|x_{k+1} - x_k| \le \theta$, where – given error of calculations; that a point with coordinates (i, x^*) is labeled on a contour preparation, where $x^* = x_k$.

Step 6. If the condition of *step 5* is not executed, the following iteration is made, and the *steps* 2 – 5 are repeated.

Step 7. Number of a row is increased by one and the algorithm repeats before the end of processing, while $i \le i_{\max}$.

It is offered to increase the iteration speed by reducing of the convolution operations number. If $h(x)$ – pulse characteristic of the filter, and $g(x) = h(x)*h(x)$, the transformation $Wf(x)$ can be written down with the account (13)

$$Wf(x) = -f(x) * h(x), \quad W(Wf(x)) = (f(x) * h(x)) * h(x) \quad \text{or}$$

$$W(Wf(x)) = f(x) * g(x). \qquad (19)$$

Taking into account properties of Fourier transform,

$$g(x) = \Phi^{-1}[(\Phi(h(x))^2]. \qquad (20)$$

where Φ, Φ^{-1} – operators of forward and inverse of Fourier transform.

Then kvazi Newton method algorithm of edge points search in WT space (18) is possible to be presented as

$$x_{k+1} = x_k + \left.\frac{f(x) * g(x)}{f(x) * h(x)}\right|_{x = x_k} . \qquad (21)$$

The pulse characteristics of filters $f(x)$ and $g(x)$ also can be calculated beforehand. The offered simplification of calculations allows receiving a significant advantage of speed in comparison with initial algorithm.

5. Contour analysis

Contour analysis (contour description) is result of the morphological contour processing of the gray scale images with use iterative search method on a basis of WT [3]. Algorithm takes into account the following restrictions:

1. For definition of similarity of pixels, belonging to the contour, the value of the response of the WT operator in some point (x_{1p}, x_{2q}) is used which is defined as discrete convolution

$$W_1F(x_{1p}, x_{2q}) = \sum_{i=-N}^{i=N} F(x_{1p-i}, x_{2q})\psi_i ,$$

$$W_2F(x_{1p}, x_{2q}) = \sum_{i=-N}^{i=N} F(x_{1p}, x_{2q-i})\psi_i , \qquad (22)$$

where N – order of the wavelet-filter.

2. The continuity of the contour is supposed: at small – up to 3 pixels breaks each subsequent point should lay within the limits of determined, for example, circle with the size of one in relation to the found point of contour.

3. At a choice of initial approximation the meanings received on the previous step (applied iterative search methods are used).

The algorithm allows receiving the consecutive description of a contour by spatial morphological processing of the initial image [3]. Pixel of the image with coordinates (x_1, x_2) belongs to contour, if it is similar to a point (x_{10}, x_{20}), already belonging to edge

$$\Delta WF(x_1, x_2) = \left|WF(x_1, x_2) - WF(x_{10}, x_{20})\right| \le E , \quad (23)$$

where $WF(x_1, x_2) = \sqrt{W_1F(x_1, x_2)^2 + W_2F(x_1, x_2)^2}$;

$WF(x_{10}, x_{20}) = \sqrt{W_1F(x_{10}, x_{20})^2 + W_2F(x_{10}, x_{20})^2}$;

E – some constant (threshold value).

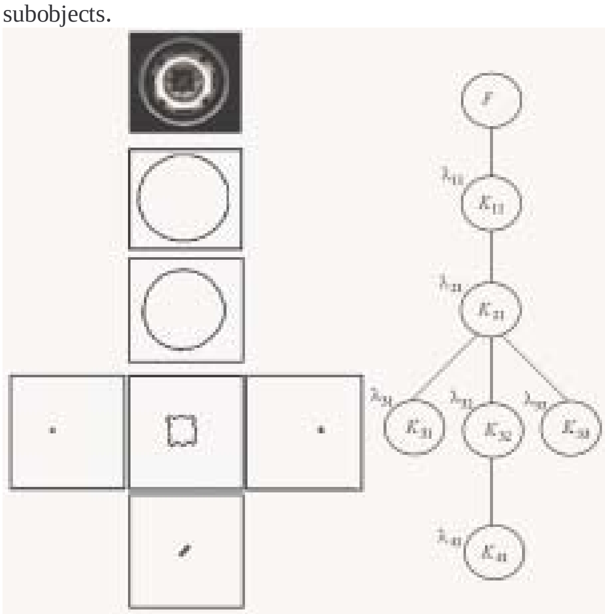
In this case morphological contour linkage points operator X for the gray scale images looks like:

$$\mathbf{X}_k = (\mathbf{X}_{k-1} - b)\left|\{WF(\mathbf{X}_k) = \right.$$

$$= \max(WF(\mathbf{X}_{k-1} - b)) \,\&\, (\Delta WF(\mathbf{X}_k) < E); b \in \mathbf{B}; \mathbf{X}_k \in D_F\} \; (24)$$

where b – displacement; B – n-component mask (primitive); D_F – definition range of the image F.

At the fig. 1 the result of the contour analysis of gray scale image is shown. There are contour preparation (fig. 1, a) and the tree of contour description (fig. 1, b). Under the received descriptions of objects K (2) the vectors of features $\mathbf{C}^j_{IKO}$ are calculated. The transition to patterns formation model considering the semantic importance (4) allows to do both an independent classification on the basis of patterns features vectors $\mathbf{C}^j_{IKO}$ and their semantic importance, and to take into account spatial ratio between patterns of objects and subobjects.



a b

Figure 1: Result of the contour analysis of gray scale image at WT space: contour preparation (a); the tree of contour description (b)

6. Conclusions

The developed of patterns formation model of hierarchical structure objects at recognition of the gray scale images takes into account both geometrical features of object, and semantic importance of its components. The realization of such model is possible with application of wavelet transform, that have properties of frequency-spatial selectivity, allowing to detect information elements of object (for example, contour) with required detailed elaboration. The morphological operator allowing receiving the contour description at different hierarchy levels is developed on WT base. Use of patterns formation model considering the semantic importance allows avoiding necessity of repeated comparison of the received information with the large number of the standards and, therefore, to raise acceptance speed of the classification decisions at recognition of hierarchical structure objects and analysis of scenes.

7. References

[1] Pratt W.K. *Digital image processing. – 3nd ed. – USA: John Wiley & Sons, 2001.*

[2] Антощук С., Крилов В., Бабілунга О. *Ієрархічна модель контурної сегментації зображень* // Праці VIII Всеукраїнської міжнарод. конф. “УкрОБРАЗ”. - Кіїв: вид. МНВЦІТС, 2006. - С.107-110.

[3] Антощук С.Г., Клих Ю.А., Николенко А.А., Бабилунга О.Ю. *Обоснование метода поиска экстремума при наличии помех с использованием гиперболического вейвлет-преобразования* // Зб. наук. праць / Одеський ордена Леніна інститут Сухопутних військ. – Вип. 13 (ч. 1). – Одеса: ООЛІСВ, 2007. – С.5-11.