Nearly-Perfect-Reconstruction Filter Banks with Critical Sampling: Achievable Quality with Quantized Coefficients

Tomasz Ruść *

University of Kiel
Inst. for Circuits and Systems Theory
Kaiserstr. 2, D-24143 Kiel, Germany
tru@tf.uni-kiel.de

Ulrich Heute

University of Kiel
Inst. for Circuits and Systems Theory
Kaiserstr. 2, D-24143 Kiel, Germany
uh@tf.uni-kiel.de

Abstract

For cosine-modulated filter-bank systems with critical sampling and near-prefect reconstruction, three prototype-design methods were compared, recently, in forms of their achievable quality. Now, the influence of coefficient quantization is investigated: By means of extensive Matlab simulations, filter banks with different channel numbers M, prototype-filter length N, and coefficient wordlengths w_c are analyzed with regard to their quality.

1. Introduction

Filter banks are very popular in a number of signal processing applications (e.g. sub-band coding, sub-band adaptive filtering). Cosine-modulated filter banks have been of special interest because of their simplicity and efficient implementations in polyphase structure. Due to the progress in electronics, hardware solutions have become feasible which employ filter banks with large "channel" numbers $M = (...16...32...64...128...256...)$. Due to cost and power consumption, however, fixed-point implementations are still favorable. While such implementations of filter banks on a signal processor with fixed-point arithmetic allow fast and efficient computation, they require to quantize the filter coefficients of the designed prototype filter. This, however, changes the reconstruction behavior in an analysis-synthesis system: If, ideally, the system would provide perfect reconstruction (PR), this property will, in general, be lost, and in near-PR (NPR) cases, aliasing and linear distortion will increase. There are deep investigations on quantization effects in FIR filters (see, e.g., [13], [10],[9]) and some on those in filter banks [12]; there are even PR designs assuming integer coefficients in filters and transformations ([6], [11]). Nevertheless, little is known on the NPR case which should provide more flexibility, usable for specific application aspects.

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In the following, NPR cosine-modulated systems with critical sampling and identical filters on both sides are addressed. Three design procedures are compared to each other with respect to the achievable quality in terms of linear and aliasing distortions, and, especially, the impact of coefficient quantization is analyzed. The aim is a statement on filter length n_F and word length w_c needed for a prescribed system quality.

2. Uniform Cosine-Modulated Filter Bank

A typical analysis-synthesis cosine-modulated filter bank has two sets of M filters, namely, an analysis filter bank and a synthesis filter bank. All impulse responses of the

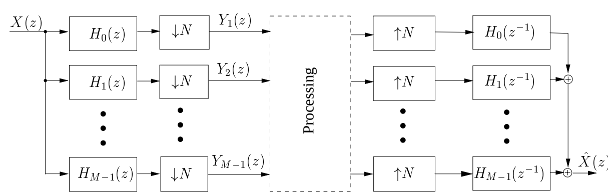

Figure 1: *Direct form realization of analysis and synthesis filter bank.*

analysis filter bank are derived from a lowpass prototype filter with coefficients $p(n)$ as

$$h_k(n) = 2p(n) \cdot cos((k + \frac{1}{2})(n - \frac{L-1}{2})\frac{\pi}{M} + (-1)^k \frac{\pi}{4}) \quad (1)$$

The M analysis filter transfer functions are then formed according to

$$H_k(z) = \sum_{l=0}^{L} h_k(n) z^{-n} \qquad 0 \leq k \leq M-1 \quad (2)$$

Likewise, the M synthesis filters are formed according to

$$G_k(z) = \sum_{l=0}^{L} g_k(n) z^{-n} \qquad 0 \leq k \leq M-1 \quad (3)$$

where the impulse responses of the synthesis filter bank are also modulated as

$$g_k(n) = 2p(n) \cdot cos((k + \frac{1}{2})(n - \frac{L-1}{2})\frac{\pi}{M} - (-1)^k \frac{\pi}{4}). \quad (4)$$

Here, $k = 0, 1, 2...M - 1$ denotes the channel number, and L the numbers of coefficients of the prototype-filter impulse response. Each subband signal is decimated by a factor N. An implementation of this filter bank is depicted in Fig. 1. From Fig. 1, the input-output relationship can be derived as

$$\hat{X}(z) = \frac{1}{N} \sum_{l=0}^{N-1} X(zW_N^l) \sum_{k=0}^{N-1} H(zW_M^k W_N^l) H^*(zW_M^k) \quad (5)$$

where $W_N = exp^{\frac{-j2\pi}{N}}$ and $W_M = exp^{\frac{-j2\pi}{M}}$, and '*' denotes conjugation. This may be rewritten as

$$\hat{X}(z) = \sum_{l=0}^{N-1} A_l(z) X(zW_N^l) \quad (6)$$

with

$$A_l(z) = \frac{1}{N} \sum_{k=0}^{M-1} H(zW_M^k W_N^l) H^*(zW_M^k). \quad (7)$$

If $A_l(z) = 0$ for $l = 1, 2, ...N - 1$, and $A_0(z) = az^{-b}$, for any a, b where $a \neq 0$, we get a perfect-reconstruction filter bank. However, any filtering operation in the subband or quantization of coefficients of the prototype filter may cause possible phase and amplitude changes, and the PR property will be lost. Filter banks which allow a small amount of aliasing and /or small linear distortions are called “near-perfect-reconstruction” (NPR) systems. Their quality can be measured by the maximum linear distortion (LD_{max}), the maximum alias distortion (AD_{max}), the mean-square amplitude distortion (termed ”energy E_1”), and the mean aliasing power proportional to the mean-square stopband attenuation (called ”energy E_2”). The definitions of these quality parameters can be found in [1]. Several algorithms for the design of NPR filter-bank prototypes were proposed, such as those due to Xu [4], Kliewer [2], and Nguyen [3]. These three were briefly described and investigated, without regarding quantization, in [1]. Here, we shall analyze the influence of coefficient quantization on the performance of the NPR system.

3. Coefficient-Quantization Impact

3.1. Quality Decrease

Assuming that any design of the prototype with “computer accuracy” gives, in an appropriate sense, the “optimum” possible quality, the conclusion that coefficient quantization reduces the quality is quite obvious. In [1], relations between the quality-describing parameters and the prototype and filter-bank features were given. Now, the aim it to find corresponding relation between the quality and prototype-coefficient wordlength w_c.

3.2. Zero Coefficients

A quick glance to some typical examples shows already that, after quantization of the prototype-filter coefficients, numerous coefficients are rounded to 0. Since, at least for some prototype designs, a typical low-pass impulse response will be similar to a sinc-function, decaying proportionally with $\frac{1}{n}$, especially high-order filters will have many very small coefficients in the ”tails” of $h_0(n)$, leading to many zeros after quantization. So, one receives filters with effectively shorter impulse responses and worse output parameters (wider transition bands, narrower passbands, bigger deviations in passband and stop-band). The influence of quantization on the length of the impulse response of the prototype filter with varying M and w_c is shown in Fig.2. The first observation is that quantized

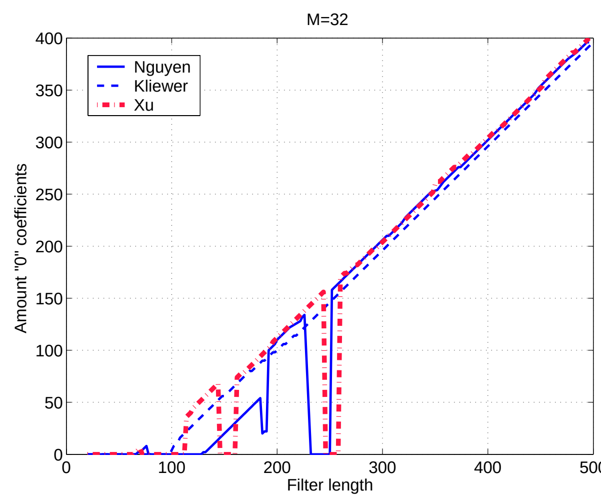

Figure 2: *Number of zero coefficients of even-length prototype filters for a word-length $w_c = 8$ bits.*

prototype filers, designed by Kliewer’s algorithm (dashed line) have an almost linear relation between the number of zero coefficients and the original filter length. The design methods proposed by Nguyen and Xu do not have this property, unless the order N is very high. This is related to the fact that Kliewer’s algorithm makes a modification of the characteristic in the frequency domain and the prototype filter coefficients are calculated from an IFFT, which leads to a sinc-type behavior. The other two methods are using time-domain constraints, and therefore they modify the filter impulse response which finally does not approximate any “ideal sinc” shape.
Based on this observation, we tried to relate the number of vanishing coefficients directly to the quality loss due to the limited wordlength w_c.

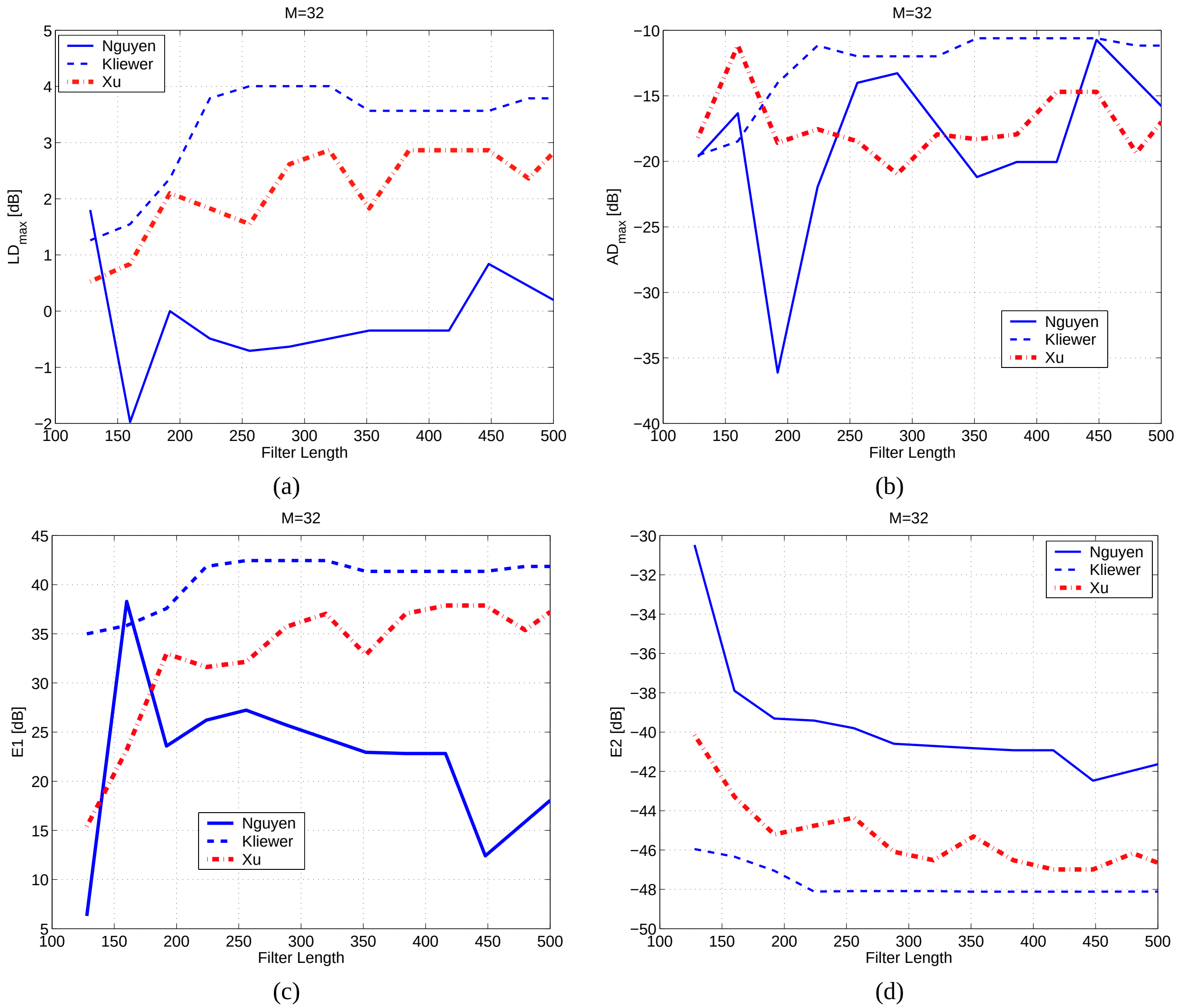

Figure 3: *Comparison of 3 methods to design cosine-modulated filter banks for quantized prototype filters with* $w_c = 8$ *bits. (a) Maximal linear distortions. (b) Maximal alias distortions. (c) Energy* E_1*. (d) Energy* E_2*.*

This leads to some insight indeed, but not to a closed formula. The reason is quite simple: As all error contribution enter $H(z)$ or $G(z)$ in a linear way, quantization to zero is in no way more important than that to any other interval representative.

3.3. Quality Limitation due to Quantization

Fig.3 shows, as an example, the quality parameters for a 32-channel filter bank with the prototype coefficients quantized with $w_c = 8$ bits.
The maximum errors show oscillations, as observed without quantization already [1]. Fluctuations are increasing for shorter wordlengths; this is shown in Fig.4. Therefore it is not easy to generate an estimation formula. More predictable are the averaged error measures E_1 and E_2. Obviously, for Kliewer's and Xu's designs the quality is more or less limited for $N \geq 200$. This may be explained by the observation from sec. 3.2: Above $N = 200$, there are only some 100 coefficients effective, due to many zeros, and only their variations contribute to some improvement.

3.4. Efficient Choice of Order and Word Length

In Fig.(4), polynomially smoothed variations of the maximal linear distortions are depicted, for a system with $M = 32$ channels. It is to be seen that for wordlengths $w_c \geq 11$, a quality saturation is reached, and a filter order of $N \approx 320 = 10M$ is appropriate. The same observation was made for $M = 8$.
If for some reason, $w_c \leq 9$ is needed, the best order decreases: As seen before, additional coefficients are not

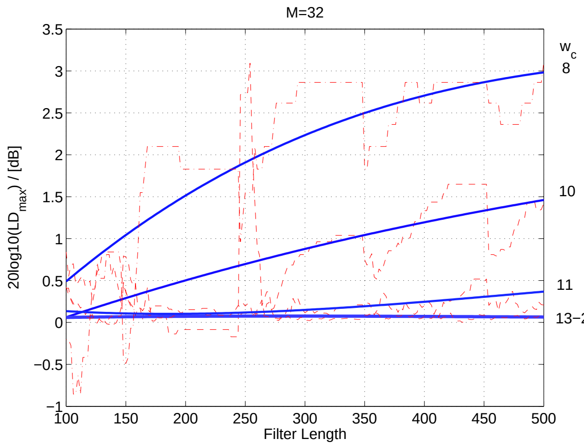

Figure 4: *Maximal linear distortions for even length of prototype filters quantized with different wordlengths w_c, for 32 channels designed by Xu's algorithm. Thin dashed lines are from simulation and thick lines are approximations by third-order polynomials.*

too helpful.

For cosine-modulated filter banks the edge frequency of the passband is defined as $f_g = \frac{\pi}{2M}$. Thus for such systems, with a large number of channels and a small value of f_g, the prototype filter is more sensitive to quantization.

4. Conclusions

In this letter, the problem of quantization of prototype-filter coefficients in an NPR filter bank has been studied. The quantization effects have been investigated for different design methods. It turns out that among the methods that have been studied, there does not exist a simple relation between the number of zero coefficients in the quantized impulse response and quality parameters of the filter bank. The maximum linear and alias distortions have high oscillation, whose amplitude is increasing for smaller wordlengths w_c. Therefore it is difficult to create a closed formula to estimate their value. For averaging error measures, however, closed formulae are worked on, with the possibility mentioned already in [1], namely, expressions for upper and lower bounds.

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