Comparison of Design Methods for Near-Perfect-Reconstruction Filter Banks with Critical Sampling

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Abstract

Cosine-modulated near-perfect-reconstruction filter banks with critical sampling can be designed by various methods. Three approaches are compared with respect to the achievable quality, as described by the following quality parameters: maximal linear distortion, maximal aliasing distortion, mean-square linear distortions, mean-square aliasing distortions. Based on extensive simulations, new formulae for the estimations of the quality from filter bank parameters are derived. Problems in special cases are discussed.

1. Introduction

1.1. Analysis-Synthesis Filter Banks

Systems with an analysis and a synthesis filter bank (see fig.1) are used in many applications: A separation of frequency components $X_M(k)$ of some signal $x(k)$, their modification/manipulation in the "frequency domain", and a re-synthesis of the modified components $\tilde{X}_M(k)$ are employed to create an output signal $\tilde{x}(k)$ which, in some prescribed sense, approximates a desired version $d(k)$ of the input.

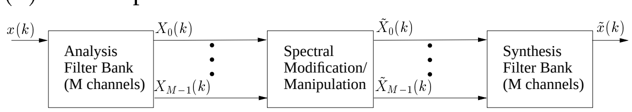

Figure 1: *Analysis-synthesis filter bank in a general application.*

Popular examples include frequency-domain coding, where a close replica of $x(k)$ is wanted, i.e.

$$\tilde{x}(k) \overset{!}{\approx} d(k) \doteq x(k) \qquad (1)$$

with as few bits as possible allocated to different, adaptive quantization of the spectral values $X_\mu(k)$, or echo and/or noise-reduction techniques where $\tilde{x}(k)$ should approximate a "cleaner" version $d(k)$ of the disturbed input $x(k) = d(k) + n(k)$, with $n(k)$ denoting some additive distortion signal. There are various approaches for the prototype design (see, e.g., [1]-[4]). Some of them aim at "perfect reconstruction" (PR), where, without any intermediate modification, i.e., for $\tilde{X}_\mu(k) \equiv X_\mu(k) \forall \mu$, also

$$\tilde{x}(k) = A \cdot x(k - k_0) \qquad (2)$$

holds; A is a real constant factor, and k_0 is a constant integer delay. As in many applications, however, the output approximation $\tilde{x}(k) \overset{!}{\approx} d(k)$ is far from being perfect anyway (as in low-rate coding or in some SNR enhancement from, e.g., $SNR_x \approx 0dB$ to $SNR_{\tilde{x}} \approx 15dB$), a "nearly-perfect" reconstruction (NPR) suffices, replacing (2) by

$$\tilde{x}(k) \approx A \cdot x(k - k_0) \qquad (3)$$

and allowing for both some linear distortion and some aliasing error in the output signal.

In the following, NPR cosine-modulated systems with critical sampling and identical filters on both sides are addressed. Several design procedures are compared to each other with respect to the achievable quality in terms of linear and aliasing distortions, and, especially, the impact of non-qantized prototype filter coefficients is analyzed. The aim is a statement on the filter length n_F and the number of channels M needed for a prescribed system quality. The estimation formulae presented in this paper are the first part of our research work. A second part, based on these results, will be dedicated to point out the influence of coefficient quantization [8].

2. NPR Design Procedures for Critical Sampling

2.1. Choice of Approaches

This section presents 3 different design methods of criticall sampled cosine-modulated filter banks with NPR property. We have chosen algorithms which guarantee a stable design process for varying prototype filter lengths while having different levels of complexity.

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The first approach, proposed by Kliewer [1], is based on [6], but uses an improved frequency-sampling design, where the desired frequency response is constructed such that it can be represented by a linear-phase FIR filter. The algorithm begins with designing a Nyquist filter with impulse response $h(n)$ and frequency response $H(e^{j\omega})$. Then the following constraint (4) is approximated in a least-squares sense by modification of $H(e^{j\omega})$:

$$\left|H(e^{j\omega})\right|^2+\left|H(e^{j(\omega-\frac{2\cdot\pi}{2\cdot M})})\right|^2 = \begin{cases} 1 & 0 \le \omega < \frac{2\pi}{2M}, \\ 0 & \omega < -\frac{2\pi}{2M}, \omega \ge \frac{4\pi}{2M} \\ arbitrary\ elsewhere \end{cases} \tag{4}$$

An IDFT of the sampled modified frequency response yields the impulse response $h(n)$ of the desired prototype filter.

The next algorithm, proposed by Xu et al. [3], is an extension of the design of 2-channel non-modulated filter banks as mentioned in [5] (frequency domain approach) and [4] (time domain approach). The design problem is reduced to the problem of minimizing the object function

$$E = E_1 + \alpha \cdot E_2, \tag{5}$$

where

$$E_1 = \int_0^{\frac{\pi}{M}} [|H(e^{j\omega})|^2 + |H(e^{j(\omega-\frac{\pi}{M})})|^2 - 1]^2 d\omega \tag{6}$$

is the mean-square amplitude distortion (termed "energy E_1") of the designed filter bank, and

$$E_2 = \int_{\omega_s}^{\pi} |H(e^{j\omega})|^2 d\omega \tag{7}$$

desribes the mean-square stop-band error (termed "energy E_2") of the prototype-filter frequency responce $H(e^{j\omega})$, where $\omega_s = \pi/(2\cdot M) + \epsilon$, and ϵ is a positive constant which defines the required transition width. The prototype filter $H(e^{j\omega})$ is a linear-phase lowpass filter with a symmetrical impulse response, its frequency response can be expressed as

$$H(e^{j\omega}) = H_0(\mathbf{h},\omega)e^{-j\omega(N-1)/2} \tag{8}$$

where $\mathbf{h}$ is the coefficient vector, and

$$H_0(\mathbf{h},\omega) = \sum_{n=0}^{\frac{N}{2}-1} 2\cdot h(n)\cdot cos[(n-\frac{N-1}{2})\cdot\omega] \tag{9}$$

for an even filter length N, and

$$H_0(\mathbf{h},\omega) = h(\frac{N-1}{2}) + \sum_{n=0}^{\frac{N-1}{2}-1} 2\cdot h(n)\cdot cos[(n-\frac{N-1}{2})\cdot\omega] \tag{10}$$

for N odd. Therefore, E_1 in (6) and E_2 in (7) can be rewritten as

$$E_1 = \int_0^{\frac{\pi}{M}} [|H(\mathbf{h},\omega)|^2 + |H(\mathbf{h},\omega-\tfrac{\pi}{M})|^2 - 1]^2 d\omega,$$

$$E_2 = \int_{\omega_s}^{\pi} |H(\mathbf{h},\omega)|^2 d\omega. \tag{11}$$

The influence of both energies is controlled by the weigthing factor α ($\alpha \in [0,1]$). The function E is not minimized directly, but by an iterative procedure. At the begining of the iterative procedure, $H(\mathbf{h},\omega)$ is first designed using the Remez algorithm. A linearization is obtained by the following splitting of the squared zero-phase response:

$$[H(\mathbf{h},\omega)]^2 \approx H(\mathbf{h}_{i-1},\omega)\cdot H(\mathbf{h}_i,\omega), \tag{12}$$

where $i = 0,1,2,3,..$ is an iteration index.

The third algorithm, proposed by Nguyen [2], is based on the assumption that the k-th channel synthesis filters $G_k(z)$ and analysis filters $H_k(z)$ can be derived by modulation of a linear-phase prototype filter $P(z)$ according to:

$$H_k(z) = a_k c_k P(zW_{2M}^{(k+1/2)}) + a_k^* c_k * P(zW_{2M}^{-(k+1/2)},)$$

$$G_k(z) = a_k^* c_k P(zW_{2M}^{(k+1/2)}) + a_k c_k * P(zW_{2M}^{-(k+1/2)}) \tag{13}$$

where $0 \le k \le M-1$, $a_k = e^{j\theta_k}$, $c_k = W_{2M}^{(k+1/2)(N-1)/2}$ and $\theta_k = (-1)^k\pi/4$. The prototype of length N is designed such that $P(z)$ is a spectral decomposition of a 2M-th band filter $F(z)$ so that

$$F(z) = z^{N-1}P(z)\tilde{P}(z) = P^2(z) \tag{14}$$

is fullfiled. Under this condition the overall transfer function $T(z)$, defined as

$$T(z) = \frac{1}{M}\sum_{k=0}^{M-1} H_k(z)G_k(z) \tag{15}$$

is a simple delay. The design method finds the prototype filter $P(z)$ such that (14) is fullfiled and, additionally, describes a filter with high stop-band attenuation. Therefore, the 2M-th band filter $F(z)$ is described by means of the filter-coefficient vector $\mathbf{p}$ with elements

$$p(n) = \begin{cases} n = 0...N/2-1, & for\quad N\quad even; \\ n = 0...(N-1)/2 & for\quad N\quad odd; \end{cases} \tag{16}$$

using the symmetry of the impulse response leads to

$$F(z) = \mathbf{p}^t(\sum_{n=0}^{2\cdot N-2} z^{-n}\mathbf{D}_n)\mathbf{p}, \tag{17}$$

with a suitably chosen matrix $\mathbf{D}_n$ [2]. For the impulse response of a 2M-th band filter we have

$$g(n) = \begin{cases} 0, & n = N-1 \pm 2lM, l = \pm1,\pm2,... \\ \frac{1}{2M} & n = N-1 \end{cases} . \tag{18}$$

The comparison of the filter coefficients $g(n)$ according to (18) with $\mathbf{p}^t\mathbf{D}_n\mathbf{p}\ \forall n$ according to (17) leads to 2M conditions for the filter coefficients $p(n)$ of the prototype filter. If the conditions are fulfilled, we receive $P(z)$ as a spectral factor of the 2M-th band filter $G(z)$ according to (14). Additionally, a minimization of the alias components is achieved due to the condition of high stop-band attenuation, according to

$$E_2 = \sum_{k-1}^{S-1} \beta(k) \int_{\omega(k)}^{\omega(k+1)} |P(e^{j\omega}|^2 d\omega. \qquad (19)$$

Here, S is the number of stopbands, $\beta(k)$ are their relative weights, and $\omega(k)$ and $\omega(k+1)$ are the bandedges of these stopbands. The above optimization problem is solved by the Schittkowski minimization algorithm [7].

2.2. Comparison of Algorithms

The approaches described in section 2.1 were implemented in a Matlab environment and designed for filter lengths $N = 20 \div 500$, for different numbers of channels $(M = 4, 8, 16, 32)$. As quality measures of the filter bank the maximum linear distortion, maximum aliasing distortion, energy E_1 defined by (6), and energy E_2 as defined by (7) were considered.

The linear distortions are described by:

$$LD(e^{j\omega}) = \sum_{k=0}^{M-1} G_k(e^{j\omega}) H_k(e^{j\omega}). \qquad (20)$$

The aliasing distortions are described by an aliasing function:

$$AD_l(e^{j\omega}) = \frac{1}{M} \sum_{k=0}^{M-1} G_k(e^{j\omega}) H_k(e^{j\omega} e^{\frac{-j\omega l}{M}}) \qquad (21)$$

where $0 \le l \le M-1$ For every designed prototype filter, a maximal linear distortion was found considering only the worst case:

$$LD_{max} = 20log_{10}(\max_{\omega\in\Omega} \left|LD(e^{j\omega)})\right|). \qquad (22)$$

In the same sense,the maximal alias distortion was found. The quality parameters of designed prototype filters for 4 channels and even filter lengths are plotted in fig.2. The worst results are obtained using the approach proposed by Nguyen. It is very time consuming and has problems with finding the "true" optimum solution (local minima problem). Xu's algorithm gives the best quality parameters. It has the smallest values for both LD_{max} and energy E_1. The iterative design process was stable for all prototype filters and did not take as much time as Nguyen's algorithm. In Xu's algorithm it is possible to define the influence of the energy E_2 on the design process, by the parameter α in (5) (for our simulation: $\alpha = 1$). The approach proposed by Kliewer has a larger value LD_{max} and energy E_1 than the iterative algorithm. From these reasons, for further descriptions and for finding estimation formulas of quality parameters of filters banks we have chosen the method proposed by Xu.

2.3. Estimation of Achievable Qualities

As quality measures of the filter bank, all parameters described in section 2.2 were applied. We have observed that the magnitude of maximal linear distortions, energy E_1, and maximal aliasing have large oscillations. If we increase or decrease the prototype filter length in a 10% range, then the measured parameters (LD_{max}, AD_{max}, $E1$) may vary by $\pm 100\%$. The phenomenom is more visible for filter lengths shorter than $16 \cdot M$. The analysis and comparison of impulse responses of such prototypes did not provide a clear explanation for the reason. Therefore, the set of points were separated into two subsets, and for every subset an estimation formula was created. The formula (23) estimates the largest linear distortions: It estimates the most pessimistic case, which represents the maximal linear distortions a designer can expect for a given prototype filter length:

$$LD_{max} = acoth(A_1 \cdot N) - A_2, \qquad (23)$$

where N is the prototype-filter length,
A_1, A_2 are coefficients [1] estimated from a polynomial of third order [1] .
The smallest values of the maximal linear distortions are estimated by a second formula:

$$LD_{min} = B_1 \cdot N^2 + B_2 \cdot N + B_3, \qquad (24)$$

where B_1, B_2, B_3 are coefficients [1] estimated from a polynomial of third order [1]. The maximal $E1$ is estimated by the following formula:

$$E1_{max} = C_1 \cdot N^{C_2}, \qquad (25)$$

where C_1, C_2 are coefficients [1] estimated from a polynomial of third order [1]. The minimal $E1_{min}$ is estimated by the following formula:

$$E1_{min} = D_1 \cdot N^2 + D_2 \cdot N + D_3, \qquad (26)$$

where D_1, D_2 are coefficients [1] estimated from a polynomial of third order [1].
Approximation of maximal alias distortions for the most pessimistic case is presented below:

$$AD_{max} = I_1 \cdot N^{I_2}, \qquad (27)$$

where I_1, I_2 are coefficients [1] estimated from a polynomial of third order [1].
The energy E_2 does not have any high oscillations; therefore; we established only one formula:

$$E_2 = J_1 \cdot N^{J_2}, \qquad (28)$$

where J_1, J_2 are coefficients [1] estimated from a polynomial of third order [1].

[1] The values of all coefficients are available on the web at http://www-lns.tf.uni-kiel.de/staff/staff.htm

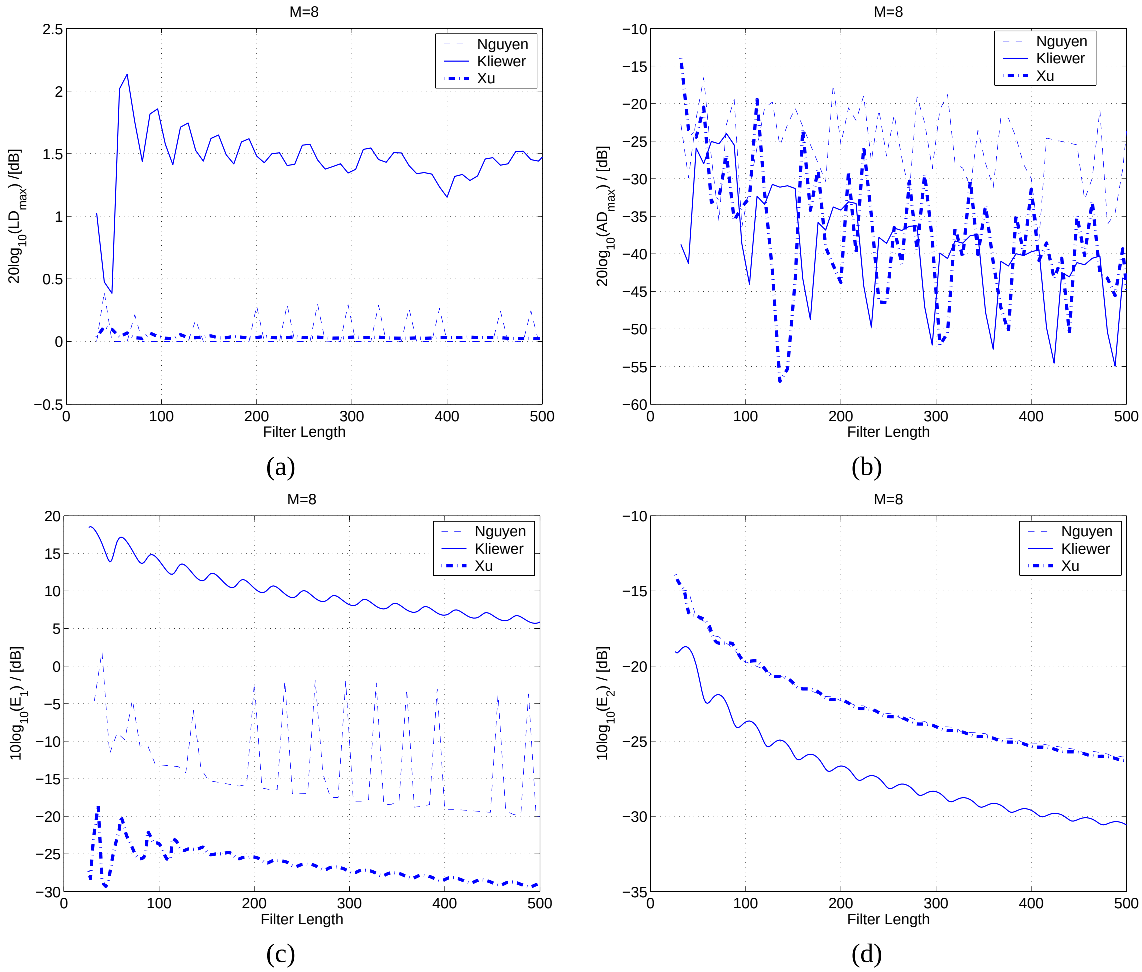

Figure 2: *Comparison of 3 methods to design cosine-modulated filter banks. (a) Maximal linear distortions. (b) Maximal alias distortions. (c) Energy E_1. (d) Energy E_2.*

3. Conclusions

In this paper, 3 algorithms to design uniform cosine-modulated filter banks have been compared. For the best approach, proposed by Xu, estimation formulas of quality parameters of filter banks have been developed. The estimation formulas will be used for further work to estimate quality parameters of prototype filters with quantized coefficients.

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