Bispectrum-based Reconstruction Technique by Tapering Pre-distortion of Image Rows

Alexander V. Totsky[a], *Jaakko T. Astola*[b], *Karen O. Egiazarian*[b], *Igor V. Kurbatov*[c], *Vladimir V. Lukin*[a]

[a] Department of Transmitters, Receivers and Signal Processing, National Aerospace University,
Chkalova Str. 17, 61070, Kharkov, Ukraine, Telephone: +38 (0572) 7074841,
E-mail: lukin@xai.kharkov.ua
[b] Tampere University of Technology, Signal Processing Laboratory, P. O. Box 553, FIN-33101,
Tampere, Finland, Telephone: +358 3 3115 2923, +358 3 3115 3860, Fax: +358 3 3115 3087,
E-mails: jta@cs.tut.fi; karen@cs.tut.fi
[c] Institute of Radiophysics and Electronics of National Academy of Sciences of Ukraine, Academika Proskury Str. 12, 61085, Kharkov, Ukraine, Telephone: +38 (0572) 7446636,
E-mail: kiv@basis.com.ua

Abstract

The problem of image reconstruction in additive noise and jitter environment is considered. Bispectrum-based Fourier magnitude and phase spectra recovery from the pre-distorted digital image rows is proposed and investigated. Pre-distortion function of Gaussian shape is introduced in each image row to decrease spectral leakage, obtain continuous phase bispectrum functions and fix the locations of the bispectrum reconstructed image rows. Computer simulations are provided to demonstrate the performance of the proposed reconstruction technique. Visual inspection of the reconstructed test images illustrates de-jittering and spectral leakage decreasing in the presence of additive white Gaussian noise (AWGN).

1. Introduction

Digital 1-D signal processing techniques based on third-order spectra (bispectra) estimation have found wide applications for data filtering, signal reconstruction of unknown waveform, object classification and recognition in astronomy [1], biomedical engineering [2], radars [3], sonars [4], and others. Developments in this area are motivated by the following advantages of bispectrum-based analysis: (1) preservation of signal phase Fourier spectrum; (2) non-sensitivity of signal recovered from bispectrum to temporal or spatial shifts of original signal; (3) suppression of AWGN of unknown variance and extraction of non-Gaussian signal information. It should be noted, however, that significant AWGN suppression can be obtained under condition of the large number of observations participated in ensemble averaging.

Since the bispectrum analysis is quite robust to AWGN and preserves signal Fourier phase, it is natural to expect promising results in cases of 2-D image bispectrum reconstruction.

In fact, several bispectrum-based image reconstruction algorithms have been proposed recently [5–8]. However, the existing approaches are usually connected with the following restrictions: (1) due to signal shift invariace property, image row Fourier spectrum recovered from bispectrum corresponds to a circularly shifted row that might cause image distortions and result in problem of image row alignment; (2) signal phase Fourier spectrum recovery from bispectrum argument provides correct results only when bispectrum and Fourier phase values are within the phase main value interval limited by (-π, +π), otherwise phase discontinuities (phase wrapping) at -π and +π, phase ambiguity and image reconstruction errors arise; (3) phase unwrapping can lead to significant distortions in the presence of AWGN.

Image reconstruction technique [9] earlier proposed by us permits to overcome aforementioned phase wrapping by introducing additive pre-distortions in the form of large amplitude δ–impulses. However, this technique resulted in arising significant distortions that are mainly concentrated at the leftmost and rightmost pixels of each reconstructed image row [9]. These errors appeared due to difficulties of additive pre-distortion compensation at the final stage of data reconstruction as well as due to spectral leakage. Moreover, bispectrum estimates (BEs) obtained for multiple noisy image realizations have been employed in our previous paper [9].

In this paper we consider the algorithm that operates in the case when only one noisy and jittery image realization is available. The benefits of the approach proposed in this paper are threefold: jitter removal, spectral leakage decreasing, and phase ambiguity avoidance.

2. Problem statement

Let us consider a 2-D digital image of a priori unknown object received in a visual communication system. Suppose this image is corrupted by zero-mean AWGN (pixel distortions) and relative positions of image rows are randomly circularly shifted with respect to their true locations due to jitter influence (spatial distortions). We also assume that each k-th (k=1,2,3,…,I) image row is a real valued sequence $\{x_k^{(m)}(i)\}$ (i=1,2,3,…,I) that is observed at the digital reconstruction system input as the following m-th (m=1,2,3,…,M and $M \neq 1$ in general case) realization (m-th repeating frame)

$$x_k^{(m)}(i) = s_k(i-\tau_k^{(m)}) + n_k^{(m)}(i), \qquad (1)$$

where $\tau_k^{(m)}$ denotes *a priori* unknown random spatial shifts of the original real valued deterministic mixed phase discrete signal $s_k(i)$ (i.e., original *a priori* unknown image of the k-th row), that we want to reconstruct; $n_k^{(m)}(i)$ is the m-th realization of AWGN of unknown variance. We also assume that $\{\tau_k^{(m)}\}$ are independent and identically distributed (i. i. d.) random integers that are commonly considerably less than I.

In practice relative random displacement between adjacent rows (jitter) can be provoked by stochastic properties of telecommunication channel, mechanical raster scanning system errors as well as by data digitizing from a noisy analog image. In the latter case, synchronization pulses are corrupted by AWGN affecting the loss of "lock" in digitizing device [10]. One can say that large jitter is one of the fundamental restrictions in high speed telecommunication systems.

Notice that the considered interference model (1) is more complicated than the existing models described in [5, 6]. In these papers, to simulate spatial and pixel distortions the total sequence of 16256 samples (original image was of size 127x128 pixels) was randomly placed repeatedly in a 1-D noise frame of 16384 samples. However, an important aspect of the problem of adjacent rows de-jittering was not treated yet. Furthermore, images restored by approach stated in [5, 6] are circularly shifted and these images need manual realignment that is a quite time consuming process. Note, that this is practically inappropriate for automatic pattern recognition systems.

To alleviate these shortcomings and restrictions, the novel approach to image reconstruction (enhancement) in jitter and AWGN environment is proposed below.

3. Proposed approach

The proposed image reconstruction technique includes the following processing stages and steps.

Stage 1. Jitter compensation (de-jittering).

Step 1.1. Computation of the cross-correlation function estimates $\hat{R}_{k,k+1}^{(m)}(l)$ for each two adjacent jittery and noisy image rows (1) according to

$$\hat{R}_{k,k+1}^{(m)}(l) = \sum_{i=1}^{I} x_k^{(m)}(i) x_{k+1}^{(m)}(i-l), \qquad (2)$$

where l=1,2,3,…,I is the spatial delay index.

Step 1.2. Evaluation and storage the maximum coordinates $\{l_{\max k'}^{(m)}\}_{jittered}$ of the functions (2) as

$$\{\hat{R}_{k,k+1}^{(m)}(l)\}_{\max} \Rightarrow \{l_{\max k'}^{(m)}\}_{jittered}, \qquad (3)$$

where k'=1,2,3,…,I-1.

Step 1.3. Computations of the corrections as

$$\Delta_k^{(m)} = \{l_{\max k'}^{(m)}\}_{jittered} - l_{center}, \qquad (4)$$

where l_{center} is the row center coordinate that corresponds to the coordinates of the maximums of original adjacent row cross-correlation functions. The value l_{center} is employed as the reference value for jitter compensation.

Step 1.4. Jittery rows alignment according to the corrections (4). After jitter compensation, the expression (1) can be rewritten as

$$x_{k\,cor}^{(m)}(i) \cong s_k(i) + n_k^{(m)}(i). \qquad (5)$$

Stage 2. Spectral leakage decrease and phase wrapping avoidance.

Step 2.1. Multiplication of the primary functions (5) by some pre-distortion function that Fourier spectrum pronouncedly has no zeros and, hence, the total function magnitude Fourier spectrum does not contain zeros. As such pre-distortion, smooth Gaussian function has been chosen in the simplest case. Pre-distorted image row then can be expressed as

$$f_k^{(m)}(i) = w_{pr}(i) x_{k\,cor}^{(m)}(i), \qquad (6)$$

$$f_k^{(m)}(I-i+1) = w_{pr}(i) x_{k\,cor}^{(m)}(I-i+1), \; i \in [1, L]$$

where $w_{pr}(i)$ is the pre-distortion (tapering) function

$$w_{pr}(i) = e^{[\mu(L-i)]^2}, \qquad (7)$$

where variables $L<I/2$ and μ determine spread and slope of (7), respectively.

It should be stressed, that signals (6) will be of maximum phase signals if maximum of the pre-distortion function (7) satisfies to the following condition

$$\{w_{pr}(i)\}_{\max} >> \sum_{i=1}^{I} x_k^{(m)}(i), \text{ for each } k. \qquad (8)$$

Step 2.2. Computation of the BEs according to

$$\hat{B}^{(m)}_{f_k}(p,q) = \left\lfloor X^{(m)}_{k\,cor}(p)\otimes W_{pr}(p)\right\rfloor \left\lfloor X^{(m)}_{k\,cor}(q)\otimes W_{pr}(q)\right\rfloor \left[X^{(m)*}_{k\,cor}(p+q)\otimes W^{*}_{pr}(p+q)\right] \qquad ,(9)$$

where $X^{(m)}_{k\,cor}(...)$ and $W_{pr}(...)$ are the direct Fourier transforms of the functions (6) and (7), respectively; $\otimes$ and * denote convolution and complex conjugation, respectively; p=1,2,3,…,I and q=1,2,3,…,I are the spatial frequency indices.

Note that the role of the pre-distortion (7) is threefold: (1) to obtain better BE (9) due to spectral leakage decrease in the sense of BE bias decrease; (2) to eliminate bispectrum phase wrapping due to transform of image row to the maximum-phase signal; (3) to fix the coordinate of each k-th image row center of gravity (CG_k).

Stage 3. Bispectrum-based image row reconstruction.

Step 3.1. Image row phase and magnitude Fourier spectra recovery from BEs (9) by recursive algorithm [1].

Step 3.2. Image row reconstruction by discrete inverse Fourier transform of the row spatial Fourier spectra.

Step 3.3. Compensation of the pre-distortions (7) by multiplying of the reconstructed image rows by the function inverse to (7).

4. Simulation results

In this paper we consider the reconstruction of the 8-bit test images with sizes of IxI=256x256 pixels. In Figures 1 and 4 the noise- and jitter-free test images are shown.

AWGN with zero mean and with fixed variance of 100 was added independently to each row. Random shift $\tau^{(m)}_k$ has been simulated with fixed maximum deviation of ±15 pixels.

The first test object ("Barbara") and the second one ("Letters") corrupted by AWGN and jitter are shown in Fig.2 and Fig.5.

As can be seen from Fig.2 and Fig.5, the images are completely concealed by jitter and AWGN and it is impossible to recognize visually a priori unknown object (to percept this image).

Figures 3 and 6 illustrate the images reconstructed by the proposed technique for L=32 pixels and μ=0.065 that corresponds to $\{w_{pr}(i)\}_{\max}$=14787 (see condition (8)).

As can be seen, the reconstructed images are sufficiently cleaner than the distorted ones. The imaged scenes in Fig.3 and Fig.6 can be confidently recognized despite the slightly jagged vertical image edges.

Image row phase Fourier spectra of the original image in Fig. 4 and the corresponding pre-distorted one in Fig. 5 are shown in Figures 7 and 8, respectively. The latter plots are given for Fourier phases bounded by [-π, π].

Figure 1: Original image.

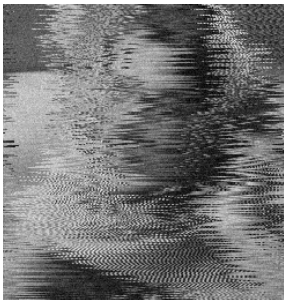

Figure 2: Image in Fig. 1 corrupted by AWGN and jitter.

Figure 3: Reconstructed image.

Figure 4: Original image.

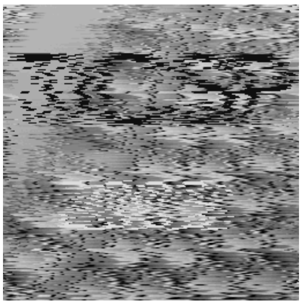

Figure 5: Image in Fig. 4 corrupted by AWGN and jitter.

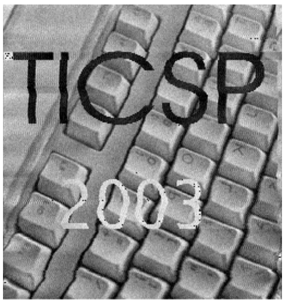

Figure 6: Reconstructed image.

One can see, that many cycles of Fourier phase wrapping are observed in Fig. 7. Due to introduced pre-distortions there is no phase wrapping in the pre-distorted row phase Fourier spectrum plotted in Fig. 8. Hence, phase errors caused by phase aliasing and wrapping can be pronouncedly decreased.

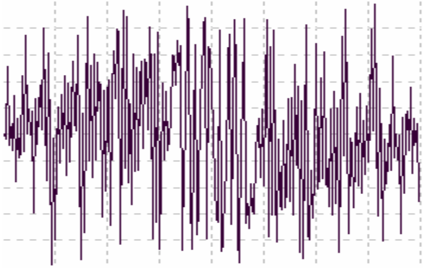

Figure 7: Phase Fourier spectrum of a noise- and jitter-free image row in Fig. 4.

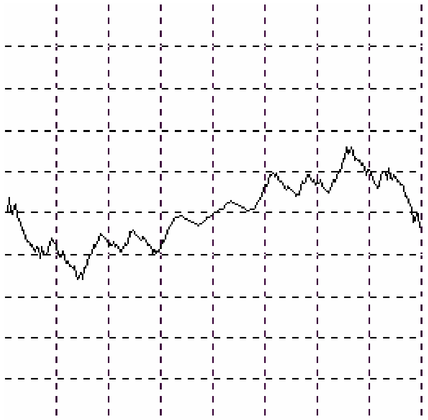

Figure 8: Phase Fourier spectrum of the pre-distorted image row in Fig. 5.

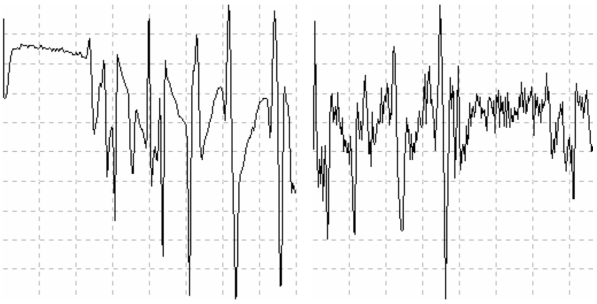

Figure 9: A noise- and jitter-free image row in Fig. 4.

Figure 10: Row reconstructed from the image in Fig. 5.

Unfortunately, there are the distortions in the reconstructed images. These distortions are caused due to the centering of the bispectrum reconstructed *k*-th image row with respect to the CG_k coordinate. If original image row was $s_k(i)$, then the reconstructed image row $\hat{s}_k(i)$ will be centered with respect to CG_k coordinate and $\hat{s}_k(i) = s_k(i - CG_k)$.

Hence, despite satisfaction of the condition (8), the CG_k coordinates of some rows may be slightly shifted relatively the central image row pixel. The residual jags of CG_k coordinate can be explained by large intensity range in different image rows.

To thoroughly study the performance of the developed technique, the original and corresponding reconstructed image rows are shown in Figures 9 and 10. As seen, the proposed technique provides good reconstructed signal quality in quite heavy jitter and AWGN environment.

5. Discussion and conclusions

Bispectrum-based technique for enhancement of a priori unknown object images corrupted by AWGN and jitter has been proposed and investigated by computer simulations. Our approach is based on jitter removal and automatic reconstructed image alignment by pre-distorting of the processed image rows. Due to introduction of the pre-distortions, only the principal arguments of the phase bispectrum are obtained. Therefore, phase unwrapping procedure is avoided and phase errors are decreased. Computer simulation results demonstrate the procedure robustness to AWGN and jitter in the case of only one image realization observed. Further reconstructed image quality improvement can be expected if more observed realizations are available.

6. References

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