Informational Assessment of Systems for Super-Resolution Restoration from Several Images

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Abstract

The method of assessment of quality of super-resolution restoration process from several images on the basis of the information theory is described. The significant advantage of the informational-theoretical approach is the opportunity of determination of total amount of information: for several imaging systems registered the same object, for multiple registration of object in different conditions of its illumination; for subpixel shift in images sequences. Simulation and modelling show that using of given method allows theoretically prediction of optimal realization of images transmission through scattering media. In the paper the results of application of quality assessment method for two problems are given. In the first, the results of restoration from subpixel shifted images sequences depending on the point spread function and numbers of shifts are investigated. In the second example, the quality assessment of real-time image restoration from several noised and smoothed by motion of object is made.

1. Introduction

Quality of isoplanatic (space invariant) imaging system can be evaluated by means of relation of Shannon information theory with minimum squared error (MSE) Wiener method of restoration [1]. In the work [1] the informational evaluation of isoplanatic optical systems, termed by amount of information in the image can be used in view of possibilities of digital enhancement of registered images. Such approach makes it possible to optimize the designing of system for image registration with the best result of restoration of images for visual perception [2].

In the nowadays imaging systems with the focal plane arrays (FPA) devices (CCD, CMOS and *etc*.) are widely used. In such a case the requirement of isoplanatism is broken (point spread function (PSF) and modulation transfer function (MTF) depends on shift of coordinates) [3]. Therefore we cannot correctly apply the theory of isoplanatic systems to the comparative analysis of quality of the modern systems. The theory of evaluation of performance of the modern optical-electronic systems is not enough. In the given paper the evaluation performance of general class of linear system is offered. For convenience practical using of the given evaluation the relation to isoplanatic analog – the amount of information in the image of Linfoot and Fellget [1] is considered. This approach has not received a wide practical spread. In our opinion this fact is stipulated by the following reasons. Than more MTF on the given spatial frequency for the given signal to noise ratio (SNR) on the same frequency the channel capacity of system is more. Therefore, the comparative analysis of isoplanatic systems in view of possibilities of digital image processing for the given SNR in practice usually is easier to carry out only with using MTF.

For informational assessment [1] of optical-electronic system on the basis FPA we should take into account the essential dependence result of restoration by Wiener method on Nyquist limit in spatial frequencies [2].

Drawback of calculation MSE of restoration in [1] can be overcome by using the linear method of restoration [4], [5]. Given method can be used for non-isoplanatic (space-variant) imaging systems, and even in cases when irregularly sampled points are used for restoration [6]. Significant feature of application of such method is the opportunity of simultaneous restoration from several images. And, as show experiments [7], the object can be restored from images which differ by physical conditions of formation. Formation conditions can have, for example, varying requirements of illumination, or using multispectral radiation, smooth, caused by nonuniform curvilinear shift during an object exposure, *etc*.

The advantage of the informational-theoretical approach is the possibility of determination total amount of information for the several imaging systems registering the same object, or for multiple recording of object under various requirements of its illumination [8]. The total amount of information calculated for process of registration of several images, or several systems of observation makes it possible to assess the dependence of restoration quality on parameters of system, PSF, illumination intensities. Also, the usage of the information performance helps to theoretically evaluate result of application of known methods [9] and [10] of super-resolution restoration from sequences noised and blurred images. Thus the amount of information for certain imaging system can be converted into performance of optical system termed MTF. For non-isoplanatic systems, application MTF theory is not correct. Therefore in this paper the definition of the equivalent MTF (EMTF) of non-isoplanatic optical-electronic system with the same amount of information as in isoplanatic systems is offered.

This paper is organized as follows. In Section 2 the mathematical model of process of image formation for the case when imaging system with FPA is given. In Section 3 the mathematical description of the theory for determination of amount of information and EMTF for such system is given. Further (Section 4) results of numerical experiments which show opportunity of the using of informational performance to two problems of restoration are given. The first one illustrates efficiency of the approach for restoration from images sequences with a low-resolution, distinguished by sub-pixel shift. The second problem shows possibility of increasing the quality of restoration of images smoothed by shift during exposure time for simultaneously usage of several images smoothed in different directions. In conclusion, problems which necessary to solve in the future are outlined.

2. Optical-electronic systems

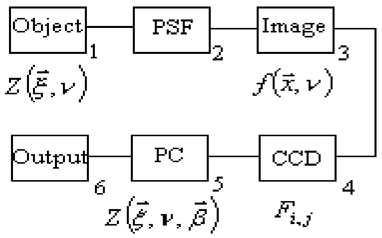


Figure 1: The model of system for image registration and processing

Let's consider the image gathering process (Fig. 1). The information about optical properties $Z(\vec{\xi},\nu)$ of object *1* is transferred by radiation through blurred media *2* with PSF and optical system *3*. The optical system *3* generates a monochromatic image $f(\vec{x},\nu)$ on a photosensitive array (*e.g.* CCD) 4 with the linear law

$$\int_{-\vec{S}}^{\vec{S}} Z(\vec{\xi},\nu) K(\vec{x},\vec{\xi},\nu) d\vec{\xi} = f(\vec{x},\nu), \quad (1)$$

where $K(\vec{x},\vec{\xi},\nu)$ the kernel of the given integrating transformation. In optics of imaging systems it is named as PSF of system, parameter ν is temporal frequency of radiation illuminating array *4*. The array *4* converts the image $f(\vec{x},\nu)$ into electrical signals $F_{i,j}$ by law (in the case of non coherence)

$$F_{i,j} = \iint_{A_{i,j}} Sen(\vec{x},\nu) f(\vec{x},\nu) d\vec{x} + \gamma_{i,j}, \quad (2)$$

where area $A_{i,j}$ integrates on photosensitive pixel with number (i, j), $Sen(\vec{x},\nu)$ is spectral function of sensitivity, $\gamma_{i,j}$ is error of electrical signal $F_{i,j}$. Combining expressions (1) and (2) we get

$$\int_{-\vec{S}}^{\vec{S}} Z(\vec{\xi},\nu) K_1(i, j; \vec{\xi},\nu) d\vec{\xi} = f_{i,j} + \gamma_{i,j} = F_{i,j}, \quad (3)$$

where $f_{i,j}$ is exact value of an electrical signal from an array device of the number (i, j), and

$$K_1(i, j; \vec{\xi},\nu) = \iint_{A_{i,j}} Sen(\vec{x},\nu) K(\vec{x},\nu) d\vec{x}. \quad (4)$$

Electrical signal $F_{i,j}$ enters the computer (the block of signals processing) *5*, which uses their values for building the continuous variant $Z(\vec{\xi},\nu,\vec{\beta})$ required performance of $Z(\vec{\xi},\nu)$. For realization this procedure we apply the algorithm, which takes into account expressions (1), (2) or (3), and uses stabilizing parameter designated here by $\vec{\beta}$. Calculated by computer $Z(\vec{\xi},\nu,\vec{\beta})$ are represented by device *6*. Quality of representation by device *6* of calculated values $Z(\vec{\xi},\nu,\vec{\beta})$ depends on many objective and subjective parameters and it is not considered in the given work. In this paper as well as in work [1] quality the system *1-5* is evaluated from positions of information theory but with usage of Kharunen-Loeva expansion instead of decomposition on system of eigenfunctions of kernel of the equation (1). Such approach takes into account non-isoplanatic devices of optical-electronic systems.

3. Theory of informational assessment

Quality of optical-electronic systems in this paper is defined by the maximum possible precision of restoration of object from electrical signals $F_{i,j}$. It is necessary to note, that problem of definition of the continuous object from a finite number inaccurately known discrete values of the right part of the equation (3) is ill-posed problem [4], and can have a set of different solutions. Therefore the additional information is necessary to check precision of its solutions by any method. The probability theory created for solutions of undetermined problems allows the calculation of their probability characteristic. For its application the object $z(\xi,\nu)$ and the error $\gamma_{i,j}$ consider as random with *a priori* given statistical characteristic values. If random object and errors are characterized by density distribution of probability, then we can to calculate density distribution of probability of object restoration error by any regularizing method [4], [5] including a nonlinear [11]. In this case it is possible to specify the task: from a set of regularized methods we have to find such one for which the probability of random object error to exceed the predefined value is minimal. Let for every spatial frequency of Fourier spectrum of object an user presets the minimal precision $a(\omega)$. Than we can to define the event

$$A(\omega,\vec{\beta}) \equiv : \{ \ \left| Z(\omega,\vec{\beta}) - Z(\omega) \right| < a(\omega) \ \}, \quad (5)$$

which means, that precision of restoration of realization of object by certain of some regularized method $\vec{\beta}$ for given spatial frequency ω is satisfactory for user. Eigenvalue of amount of information in additional event

$$\overline{A}(\omega,\vec{\beta}) \equiv : \{ \ \left| Z(\omega,\vec{\beta}) - Z(\omega) \right| \geq a(\omega) \ \} \quad (6)$$

depends on restoration method, marked here by $\vec{\beta}$, statistical *a priori* information about random noise γ_{ij} and object $z(\vec{\xi},\nu)$, from required by user precision $a(\omega)$, and from function K_1. Amount of information is found as

$$I(\omega,\vec{\beta}) = -\ln P\left[\overline{A}(\omega,\vec{\beta})\right], \quad (7)$$

where $P\left[\overline{A}\right]$ is probability of event $\overline{A}$. Obviously, that expression (7) is the natural performance of quality of system *1-5* (Fig. 1) for application of the arbitrary restoration method $\vec{\beta}$. However another performance is more interesting for optimization of process of designing the optical-electronic systems

$$I(\omega,\vec{\beta}_o) = \max_{\beta} I(\omega,\vec{\beta}) = -\ln P\left[\overline{A}(\omega,\vec{\beta}_o)\right]. \quad (8)$$

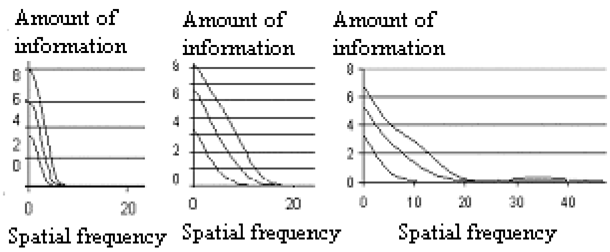
Here the maximum takes from the given set of all regularized restoration methods $\vec{\beta}$. Usually, solution of problem in real-time, the set of restoration methods is specified in parametric view. The approximated solution of the equation (1) or (3) is represented as equation, depending on one or several values of parameters $\vec{\beta}$. It is satisfy the results for the user as often as it is possible. More simply values $P\left[\overline{A}\left(\omega, \vec{\beta}_o\right)\right]$ and optimal operator $\vec{\beta}_o$ are determined, if to assume, that coefficients of random object decomposition on system of eigenfunctions of the equation (1) are statistically independent. Then the functional $\vec{\beta}$ is represented by set of monotonic functions. For this case in work [5] the expression for probability density function of restoration error on the given spatial frequency is represented depending on unknown monotonic function. Search of this function was carried out numerically from requirement $\min P\left[\overline{A}\left(\omega, \vec{\beta}\right)\right] = P\left[\overline{A}\left(\omega, \vec{\beta}_o\right)\right]$ with using of obvious relation from [5]. Two sets of functions were explored set of symmetric functions $\vec{\beta}_{lin}$ and a set of monotone functions $\vec{\beta}$. It is shown, that under a natural requirement of major uncertainty in coefficients of object decomposition, when their function of distribution of density of probabilities considerably differs from delta functions. The minimum from the set of linear functions exceeds minimum from chosen with using of the additional statistical information about random object and noise so to minimize value $P\left[\overline{A}\left(\omega, \vec{\beta}\right)\right]$. Thus, values of parameters are chosen so that all monotonic functions differs a little, namely

$$\frac{P\left[\overline{A}(\omega, \beta_{0lin})\right] - P\left[\overline{A}(\omega, \beta_0)\right]}{P\left[\overline{A}(\omega, \beta_{0lin})\right]} \le 0.15. \qquad (9)$$

Thus, separated computing results, and mainly used in practice the incomplete statistical information about object and noise as their correlation functions of the second order determine actuality of information assessment of optical-electronic systems with using of set of the linear restoration methods.

4. Examples of informational characteristic

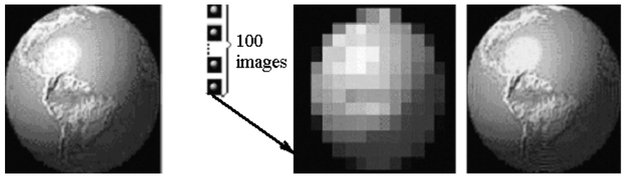
4.1 Restoration from images sequence



(a) (b) (c)

Figure 2: The dependence of amount of information on blurs. Upper curves correspond to 100 images, middle to 10 and bottom to one for cases of the heavy (a), mean (b) and small (c) blurs

The problem of restoration of object from its several blurred images was numerically simulated. The analysis of behavior of the information in dependence on number of images and PSF is carried out (Fig. 2). Sensitivity of pixel of focal-plane array was considered as the uniform and array fill factor is 50%. The period of array is dimensionless and has taken as 0,08. Spatial frequency is also dimensionless.



(a) (b) (c) (d)

Figure 3: Result of restoration. (a) - initial image; (b)- images received from (a) by shift, blur and sampling; (c)- enlarged image (b); (d) - restored from 100 images.

In Fig. 2 the dependence of the amount of information on spatial frequency for cases of the heavy (a), mean (b) and small (c) blurs (half widths of PSF are about 10 pixels, 1 pixel and 0,23 of size of pixel respectively) of the initial one-dimensional image is shown. When blur is heavy the increasing of number of sampled points in the image gives enhancement of results and SNR only on low spatial frequencies [12]. Results of two-dimensional images processing are shown in Fig. 3. The considerable improving of visual quality occurs when we use a great number of images. In this case the object is the picture 256x256 pixels, blurred images were received as images 12x12 pixels.

4.2 Restoration of smoothed images

4.2.1. Theory of restoration from several smoothed images

Let's consider the problem of restoration of object from its several images smoothed in different directions, in particular, from two-four images smoothed in x and y axes (Fig. 6, (c, d)). Initial object we can represent as:

$$Z(\xi,\eta) = \sum_p \int_{-R}^{R}\int_{-R}^{R} F_p(x,y) Q(x-\xi, y-\eta)\,dx\,dy \quad, p = 1,2,3,4. \qquad (10)$$

For many cases of processing of smoothed images it is necessary to have the algorithm, which works with high speed (in real-time mode). Therefore the "sliding window" algorithm has been used. Let the width of smooth is equal $2N+1$ points. Then restoration of object point $Z(\xi_i, \eta_j)$ conducts to summation with filter:

$$Z\left(\xi_i, \eta_j\right) = \sum_{p,l,m} F_p\left(x_{i+l}, y_{j+m}\right) Q\left(x_{l+N}, y_{m+N}\right) \quad p = 1,2,3,4;$$
$$l = -N,...N; \; m = -N,...N. \qquad (11)$$

The border points of object which placed apart from boundaries in a distance smaller than smooth are not processed. The approach is easily extended for the case of various width of smooth in the registered images.

In general case algorithm demands a complete set of orthonormal functions as a basis set of functions. As a rule, it is trigonometric series. However for usage of the given approximate "sliding window" algorithm the contribution odd component is equal to zero, and as certain realization the system has been selected

$$\psi_1(\xi) = 1/\sqrt{2S}, \;\; \psi_l(\xi) = \cos(\pi l \xi)/\sqrt{S}, \;\; l = 2,\ldots. \qquad (12)$$

Two-dimensional basis functions as multiplication of one-dimensional functions:

$$\psi_{lm}(\xi,\eta) = \psi_l(\xi)\psi_m(\eta) \qquad (13)$$

4.2.1. Numeric simulation

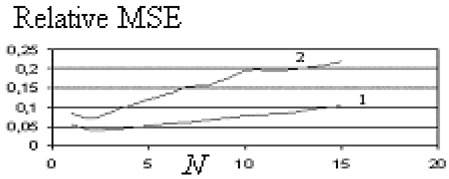


Figure 4: The dependence relative MSE of restoration by the filter 40x40 points from a half-width of smooth *N*. Curve 1 - restoration from one image, 2-from two.

For an evaluation of possibilities of restoration for the given filter and determination of required filter sizes in comparison with width of smooth the calculations MSE are performed. As shown in Fig. 4, the first curve has a flat site in field when the size of the filter approximately is twofold more than smooth size. This relation frequently used in practice. The quality of restoration from two images is considerably better, than from one (Fig. 6). For assessment of quality of restoration for such system of observation in frequency field the calculations of amount of information, statistically defining quality of processing were used. The given value shows the quality of restoration for the spatial frequencies, which correspondents to basis functions with a coefficient (l,m).

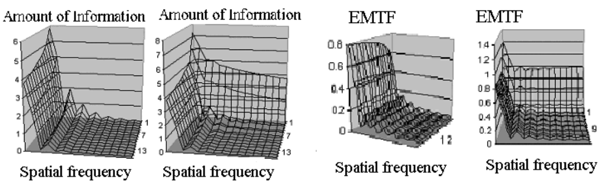


Figure 5: Amount of information and EMTF as functions from dimensionless spatial frequencies (l,m). $N=10$. (a, c) – for restoration from one image; (b, d) – from two images.

The amount of information (Fig. 5) strongly depends on coordinate of shift, thus oscillating I deals with EMTF on k frequency by relation:

$$T_k = \sqrt{\gamma^2\left(\exp\ I_k - 1\right)/\left\langle \left|c_k\right|^2\right\rangle}\,, \qquad (14)$$

γ^2 is SNR, $\left\langle |c_k|^2 \right\rangle$ - *a priori* specified values described by the kind of restored objects [6].

Behavior of EMTF in Fig. 5 (c, d) is similar to behavior of the amount of information, but it can differ by presence of large oscillations. Blurred images processing shows actual improving of visual quality for usage of two and four smoothed (with 10% of additive noise) images in Fig. 6.

5. Conclusions

The method of assessment of quality of systems for super-resolution restoration from several images on the basis of application of information theory is described. The possibility of determination of amount of information for multiply object registration is shown under its requirements of smooth and sub-pixel shift. Given information characteristic was used before [7], [8] to transfer high spatial frequency of images on low frequencies. At present the solution of problems of precise registration [13] and exact determination of sub-pixel shift in conditions of blur and noise in images are important.

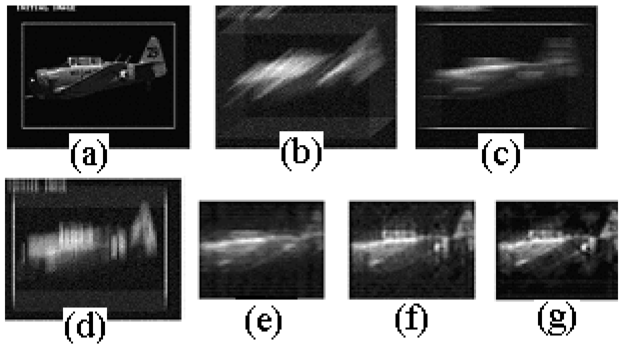


Figure 6: (a) – initial object,(b) – smoothed in diagonal direction,(c, d) – smoothed horizontally and vertically images, (e), (f) and (g) — restored from one, two and four smoothed images by “sliding window” algorithm.

6. References

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