Fidelity and correlation measures of performance of digital imaging systems

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У статті досліджуються дві методики оцінювання якості цифрових оптичноелектронних систем з фотоприймачами зображення. Перша методика базується на обчисленні середньо-квадратичного відхилення вихідних сигналів реальної та ідеалізованої оптичноелектронних систем як показника якості. Друга методика використовує коефіцієнт кореляції вихідних сигналів як показник якості.

There are several requirements to an imaging system (IS) that defines the performance - energetic, spatial and temporal resolutions [1,2]. Energetic resolution is ability to distinguish small difference of output signal amplitudes that is limited by noise. Spatial resolution is ability to differ small objects or small parts of object image where the principal limitation factor is reduction of high spatial frequencies. Temporal resolution is ability to differ fast changing of output signal amplitude when object moves in a field of view. Its limitation factor is reduction of high temporal frequencies. Generally the energetic and spatial resolutions are the most important requirements in IS design.

The key problem of IS design is to maximize the integrate parameter of IS quality called performance [1,2]. It is obvious that the performance depends on the mentioned requirements. Thus the results of design procedures will be defined by the technique of performance evaluation. Now the classical approach for performance evaluation is calculation of MRTD and probability of target discrimination using the experimental data of human vision characteristics [1,2]. This technique has become FLIR90, FLIR92 standards [1,2,3]. But it has two principal disadvantages. First, we can apply it only for IS with human observer [1]. Now many IS can operates in automatic and semi-automatic mode when an embedded does target discrimination procedures. Second, it is difficult to apply this techniques for optimization of analog-digital or analog-digital-analog IS because effects connected with analog-digital conversion like aliasing, phasing effects make dramatic influence that can not be represented by a shape of MRTD [2,3]. Summarizing said above it is clear how it is important to investigate the objective techniques for performance evaluation that consider all principal effects of analog-digital and analog-digital-analog IS with or without human observer.

The objective techniques demands an objective measure of IS quality. An IS is used for image representation, for example, an analog-digital IS represents the input optical signal in form of a matrix of digital values, analog-digital-analog IS represents it in a form of a definite realization of analog signal. Thus the objective measure of IS quality will be defined by the distortions introduced by an IS into output signal. To calculate these distortions it is convenient to create mathematical abstraction as an idealized IS. The idealized IS is a model of an IS with infinity spatial, temporal and energetic resolutions. Infinity spatial resolutions mean that spatial an temporal MTF are equal to one and a detector and electronic do not produce noise. Mathematically we can calculate the output signal of this IS that is completely repeats input optical signal. Now we can introduce performance as a measure of similarity between output signals from an investigated IS and an idealized one. Of course, there are some possibilities to calculate signal difference. The least-square error shows the absolute value of the difference, coefficient of correlation clears the difference between linear combinations of the signal and the losses of information represents the difference of amplitude distributions [4,5,6]. But all these measures have two principal advantages. First, they are objective and we can use them for IS with human observer or without him/her. Second, they allow building for IS optimization a merit function which describes energetic and spatial resolutions.

The fidelity reflects the absolute difference between the output signals of investigated and idealized IS in sense of normalized least-square error [4,5]. When the input and output signals are analog the fidelity is calculated by the expression:

$$F = 1 - \frac{\int\limits_{-\infty}^{+\infty}\int\limits_{-\infty}^{+\infty}\left(U(x,y) - U_C(x,y)\right)^2 dxdy}{\int\limits_{-\infty}^{+\infty}\int\limits_{-\infty}^{+\infty} U(x,y)^2\, dxdy} \tag{1}$$

where F - the fidelity,

x, y - the coordinates,

U(x,y), $U_C(x,y)$ - the output analog signals of the idealized IS and the real one, respectively.

In case of analog-digital IS when the output signal is digital it can be rewritten by the following ways [5]:

$$F = 1 - \frac{\sum_{i=1}^{N_X} \sum_{j=1}^{N_Y} \left(U(i,j) - U_C(i,j)\right)^2}{\sum_{i=1}^{N_X} \sum_{j=1}^{N_Y} U(i,j)^2} \quad (2)$$

where i, j - the coordinates of pixels,

N_X, N_Y - the dimensions of the output signal matrix,

U(i,j), $U_C(i,j)$ - the output signals of the idealized IS and the real one that are two-dimensional arrays of pixels, respectively. The output signal of the real IS contains two parts: the signal losses after spatial transformations including spatial filtering, sampling, digital processing - $U_{SL}(i,j)$ and noise $U_N(i,j)$ [4,5]:

$$U_C(i,j) = U(i,j) - U_{SL}(i,j) + U_N(i,j) \quad (3)$$

We accept an assumption about absence of correlation between noise and the losses $U_{SL}(i,j)$ and noise $U_N(i,j)$. Now we can rewrite (2) in the following form doing some transformations:

$$F = 1 - \frac{\sum_{i=1}^{N_X} \sum_{j=1}^{N_Y} \left(U(i,j) - \left(U(i,j) - U_{SL}(i,j) + U_N(i,j)\right)\right)^2}{\sum_{i=1}^{N_X} \sum_{j=1}^{N_Y} U(i,j)^2} = \quad (4)$$

$$= 1 - \frac{\sum_{i=1}^{N_X} \sum_{j=1}^{N_Y} U_{SL}(i,j)}{\sum_{i=1}^{N_X} \sum_{j=1}^{N_Y} U(i,j)^2} \frac{\sum_{i=1}^{N_X} \sum_{j=1}^{N_Y} U_N(i,j)}{\sum_{i=1}^{N_X} \sum_{j=1}^{N_Y} U(i,j)^2} = 1 - \left(\frac{\sum_{i=1}^{N_X} \sum_{j=1}^{N_Y} U_{SL}(i,j)^2}{\sum_{i=1}^{N_X} \sum_{j=1}^{N_Y} U(i,j)^2} + \frac{\sum_{i=1}^{N_X} \sum_{j=1}^{N_Y} U_N(i,j)^2}{\sum_{i=1}^{N_X} \sum_{j=1}^{N_Y} U(i,j)^2} \right) =$$

$$= 1 - \left(NSL^2 + NEL^2\right)$$

where NSL – the normalized spatial losses coefficient that characterizes the relationship between the signal losses and energy of an output signal of an idealized IS:

$$NSL^2 = \frac{\sum_{i=1}^{N_X} \sum_{j=1}^{N_Y} U_{SL}(i,j)^2}{\sum_{i=1}^{N_X} \sum_{j=1}^{N_Y} U(i,j)^2}$$

NEL – the normalized energetic losses coefficient that characterizes the relationship between variance of noise and energy of the output signal of an idealized IS:

$$NEL^2 = \frac{\sum_{i=1}^{N_X} \sum_{j=1}^{N_Y} U_N(i,j)^2}{\sum_{i=1}^{N_X} \sum_{j=1}^{N_Y} U(i,j)^2}$$

It is possible to apply the expression (4) for optimization considering NSL and NEL as functions of IS parameters and coefficients of a digital filter:

$$F = 1 - \left(NSL^2 + NEL^2\right) = 1 - \left(NSL^2(p_1, p_2 .. p_M, c_1, c_2 .. c_N) + NEL^2(p_1, p_2 .. p_M, c_1, c_2 .. c_N)\right) \rightarrow max \quad (5)$$

where $p_1, p_2 .. p_M$ – the parameters of an IS,

$c_1, c_2 ... c_N$ – the coefficients of a digital filter.

According to (5) optimization changes a ratio between NSL and NEL to reach an optimum balance between energetic and spatial resolutions to guarantee the highest similarity of the output signals. Note, NSL and NEL can be found for any IS without human observer. But by adding characteristics of human vision into the process of calculation of NSL and NEL it is possible to get fidelity F for an IS with human observer [1,2]. It is obvious that F=1 when an IS do not introduce any distortions into the output signal, other word when the IS acts as an idealized one. The case F=0 represents the situation when energy of the total distortions is equal to one of the output signal of idealized IS and the output signal of an IS is almost corrupted.

The coefficient of correlation between difference between the output signals of an idealized IS and a real one is an estimation of difference between its linear combinations [4]. The different gain and average levels do no influence to the correlation and only differences of signal shapes reduce it. So the coefficient of correlation is also suitable measure of similarity of the output signals. For an analog-digital IS correlation coefficient can be written by the following way:

$$r=\frac{\sum_{i=1}^{N_X}\sum_{j=1}^{N_Y}U(i,j)\cdot U_c(i,j)}{\sqrt{\sum_{i=1}^{N_X}\sum_{j=1}^{N_Y}U(i,j)^2\sum_{i=1}^{N_X}\sum_{j=1}^{N_Y}U_c(i,j)^2}} \qquad (6)$$

where $r(U,U_{SL})$ – the coefficient of correlation between the output signals from an idealized IS and the real one.

We can do several transformation of (6) to define a relationship between correlation and fidelity:

$$r=\frac{\sum_{i=1}^{N_X}\sum_{j=1}^{N_Y}U(i,j)\cdot U_c(i,j)}{\sqrt{\sum_{i=1}^{N_X}\sum_{j=1}^{N_Y}U(i,j)^2\cdot\sum_{i=1}^{N_X}\sum_{j=1}^{N_Y}U_c(i,j)^2}}=\frac{\sum_{i=1}^{N_X}\sum_{j=1}^{N_Y}U(i,j)\cdot\left(U(i,j)-U_{SL}(i,j)+U_N(i,j)\right)}{\sqrt{\sum_{i=1}^{N_X}\sum_{j=1}^{N_Y}U(i,j)^2\sum_{i=1}^{N_X}\sum_{j=1}^{N_Y}\left(U(i,j)-U_{SL}(i,j)+U_N(i,j)\right)^2}}= \qquad (7)$$

$$=\frac{\sum_{i=1}^{N_X}\sum_{j=1}^{N_Y}\left(U(i,j)^2-U(i,j)\cdot U_{SL}(i,j)+U(i,j)\cdot U_N(i,j)\right)}{\sqrt{\sum_{i=1}^{N_X}\sum_{j=1}^{N_Y}U(i,j)^2\sum_{i=1}^{N_X}\sum_{j=1}^{N_Y}\left(U(i,j)^2-2\cdot U(i,j)\cdot U_{SL}(i,j)+U_{SL}(i,j)^2+U_N(i,j)^2\right)}}=$$

$$=\frac{1-\frac{\sum_{i=1}^{N_X}\sum_{j=1}^{N_Y}U(i,j)\cdot U_{SL}(i,j)}{\sum_{i=1}^{N_X}\sum_{j=1}^{N_Y}U(i,j)^2}\cdot\sqrt{\frac{\sum_{i=1}^{N_X}\sum_{j=1}^{N_Y}U_{SL}(i,j)^2}{\sum_{i=1}^{N_X}\sum_{j=1}^{N_Y}U_{SL}(i,j)^2}}}{\sqrt{\left(1-2\cdot\frac{\sum_{i=1}^{N_X}\sum_{j=1}^{N_Y}U(i,j)\cdot U_{SL}(i,j)}{\sum_{i=1}^{N_X}\sum_{j=1}^{N_Y}U(i,j)^2}\cdot\sqrt{\frac{\sum_{i=1}^{N_X}\sum_{j=1}^{N_Y}U_{SL}(i,j)^2}{\sum_{i=1}^{N_X}\sum_{j=1}^{N_Y}U_{SL}(i,j)^2}}+NSL^2+NEL^2\right)}}=$$

$$=\frac{1-r\left(U,U_{SL}\right)\cdot NSL}{\sqrt{\left(1-2\cdot r\left(U,U_{SL}\right)\cdot NSL+NSL^2+NEL^2\right)}}=\frac{1-r\left(U,U_{SL}\right)\cdot NSL}{\sqrt{2\cdot\left(1-r\left(U,U_{SL}\right)\cdot NSL\right)-F}}$$

where $r(U,U_{SL})$ – the coefficient of correlation between the output signal from an idealized IS and the spatial losses introduced by a real IS.

The expression (7) shows how to calculate the coefficient r using fidelity F and the coefficients $r(U,U_{SL})$ multiplied by NSL which describes spatial transformations of output signal. At the same time fidelity F represents influence of noise and distortion of spatial structure of the output signal. The coefficient r as measure of similarity of the output signals can also play a role of a merit function for IS optimization:

$$r=\frac{1-r\left(U,U_{SL},p_1,p_2..p_M,c_1,c_2..c_N\right)\cdot NSL\left(p_1,p_2..p_M,c_1,c_2..c_N\right)}{\sqrt{2\cdot\left(1-r\left(U,U_{SL},p_1,p_2\,p_M,c_1,c_2..c_N\right)\cdot NSL\left(p_1,p_2..p_M,c_1,c_2..c_N\right)\right)-F}}\rightarrow max \qquad (8)$$

Now we have two merit functions: the first one (5) characterizes the absolute difference and the second one (8)– relative difference of the output signals. As a fidelity (5) the correlation coefficient can be applied for optimization of IS with or without human observer. Both of them help to find an optimal balance between energetic and spatial resolutions but the weights of these resolutions will be different.

To illustrate the proposed technique we investigate performance of a typical staring thermal imager that has diffraction limited optics with numerical aperture 1:2, photon noise limited 256 x 256 focal plane array working in 8 – 12 mkm optical range with 0.04 sec frame period. Fig.1 shows dependence F (5) and r (8) from dimensions of a photosensitive element for a four bars test-object [1,2]. This test-object according to the international standards is characterized temperature difference ΔT, background temperature T and angular period of a bar α [1,2] (Fig.1). The growth of the dimensions causes bigger spatial losses due to high harmonics reduction and it increases signal-to-noise ratio due to the bigger absorbed radiant flux. Note, that maximum position of correlation coefficient is shifted into bigger values of the dimensions because it is less sensitive to spatial distortions than fidelity (Fig.1). It is important that in case of large and contrast test-objects dependence of different measures of performance from IS parameters is rather weak. In case of small dimensions and small contrast this dependence becomes strong and depended on the performance measure.

We introduced a model of an idealized imaging system as a powerful tool for performance evaluation. Now performance is considered as a measure of similarity of output signal from idealized IS and investigated one. The first possible measure is absolute - fidelity which represents absolute difference of amplitude values. The second one is relative - correlation coefficient which reflects only changes of signal shapes. These two interconnected measures can help to maximize performance by optimization balance between energetic and spatial resolutions.

References

1. J. Howe, "Electro-Optical Imaging System Performance Prediction", *Infrared and Electro-Optical Systems Handbook.* Editor Dudzik M., Volume 4, Chapter 2, SPIE Press, Bellingham, 1993.
2. G. Holst, *Electro-Optical Imaging System Performance.* SPIE Press, Bellingham, 1995.
3. W. Wittenstein, Minimum Temperature Difference Perceived- a new approach to assess undersampled thermal imagers. // Optical Engineering – Vol.38 – No. 5 – 1999 – p.773-781.
4. W.K.Pratt, Digital Image Processing, p.685-686, John Wiley & Sons Inc., New York, 1991.
5. S.K. Park, Z. Rahman, Fidelity analysis of sampled imaging systems.// Optical Engineering – Vol.38 – No. 5 – 1999 – p.786-800.
6. F.O. Huck, R.A. Gartenberg, S.K. Park, Z. Rahman, Informatic-theoretic assessment of sampled imaging systems. // Optical Engineering – Vol.38 – No. 5 – 1999 – p.742-762.

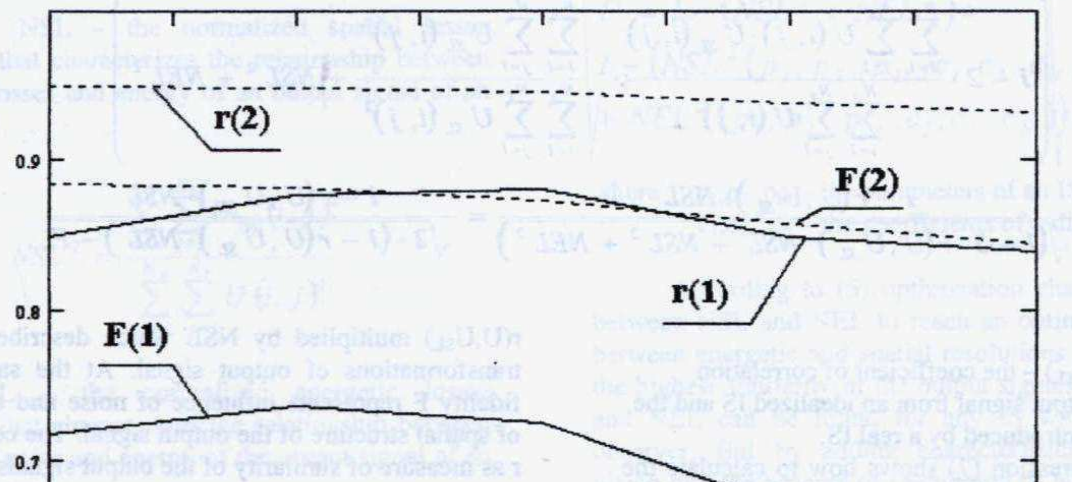

Fig.1. Relationship between the fidelity F, the correlation coefficient r and the dimension of a photo-sensitive element x (case 1 –four bars test-object with temperature difference $\Delta T = 0.05$ K and angular period of a bar $\alpha = 1$'; case 2 - $\Delta T = 0.1$ K , $\alpha = 4$', background temperature – T=293K).