ADAPTIVE MYRIAD FILTERS FOR 1-D SIGNAL PROCESSING

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ABSTRACT

The generalized adaptive myriad filter framework is proposed and the technique for such filter design is described. Within this framework and by applying two new local activity indicators, the novel versions of locally adaptive myriad filter are designed and tested. The appropriately good performance of these new filters is demonstrated for the case of intensive mixed additive and impulsive noise. The recommendations concerning the filter parameters selection are presented.

1. INTRODUCTION

For many practical applications of 1-D signal processing it is typical that informational signal is corrupted by mixed noise, i.e. simultaneously present additive and impulsive noise. For improving the extraction of useful information from signal or for enhancing the accuracy of its post-processing in these applications it is necessary to reduce the influence of that type of noise. For these purposes linear filters cannot be applied due to presence of impulses, and, furthermore, transitions in informational signal of its own. Therefore, nonlinear filters should be used since they are able to simultaneously remove spikes, suppress the additive noise component and preserve transitions in signals well enough [1].

But nonlinear filters do not possess these "contradictory" properties simultaneously. Filters can only provide some trade-off between additive noise suppression efficiency, transition preservation and impulse rejection; besides, this tradeoff for each nonlinear filter is fixed. This shortcoming can be overcome in the recently introduced myriad filter [2].

The main feature of standard myriad filter (SMF) is that it has a tunable parameter k that controls its behavior. For rather small k values the SMF works in essentially nonlinear mode and possesses good robust properties. On the contrary, when k is rather large, SMF practically performs as mean filter and, consequently, it has good additive noise suppression. Thus, for SMF (in case of the fixed scanning window size N) the trade-off between the additive noise suppression efficiency and the ability to remove spikes can be easily changed by means of varying the tunable parameter k.

Although it is possible to use non-adaptive myriad filter with appropriately chosen parameter k [3], its adaptive version is more powerful and useful [4]. The two versions of adaptive myriad filter (AMF) presented in [4] are rather simple. In the first one, the tunable parameter k for each running window position i is calculated as $k_i = k_Q \cdot Q_i^{pq}$, where Q_i^{pq} is the quasi-range (the difference between the q-th and p-th order statistics for scanning window size N) of data sample from the current running window, and k_Q is a parameter. In the second version, the parameter k is obtained as $k_i = k_{Qf} \cdot Q_i^f$, where Q_i^f is the output of the standard median filter with the same window size N applied to the array of the values of quasi-range Q_i^{pq}.

As it was shown in [4], these AMFs with appropriately chosen parameters k_Q and k_{Qf} outperform non-adaptive myriad filters. Moreover, in many cases the performance of AMFs is better than for median and α-trimmed filters (for the same N) that are widely used for 1-D signal processing. However, the determination of the parameters k_Q and k_{Qf} is a rather difficult task. Besides, in many practical situations the robustness of the proposed AMFs is not appropriate. Hence, below we propose some new versions of the AMF for which these shortcomings are overcome.

2. GENERALIZED ADAPTIVE MYRIAD FILTER FRAMEWORK

In general, the myriad filter adaptation means that for each running window position i the value of some local activity indicator (LAI) is calculated and then, taking into account the LAI value, the tunable parameter k is adaptively adjusted, thus, $k_i = f(LAI_i)$ where f is some function. As a function f we propose to use linear dependence implying some coefficient k_{LAI} since this approach was shown itself as rather effective one [4]. So, let's further suppose that $k_i = k_{LAI} \cdot LAI_i$.

Furthermore, SMF is scale non-invariant [2], i.e. its properties depend upon data scale, hence we propose to use some normalized tunable parameter k/σ_a instead of parameter k, where σ_a is the standard deviation (STD) of additive noise. So, as will be denoted below, we assume that additive component of the mixed noise obeys Gaussian distribution with the STD σ_a.

Summarizing the aforesaid, our design of the proposed AMFs consists of two stages. At the first one, an optimal value of $(k/\sigma_a)_{opt}$ that provides the best trade-off between additive noise suppression efficiency, signal transitions preservation and spike rejection abilities is found. At the second stage a coefficient $k_{SC} = \sigma_a / LAI$ that is simply the proportionality coefficient between STD and LAI values for the constant level signal fragment is determined. Finally, the k_{LAI} value is calculated as $k_{LAI} = (k/\sigma_a)_{opt} k_{SC}$.

The requirements to the LAI are the following. First, its value should characterize the scale of data samples in the scanning window that are not corrupted by spikes, i.e. LAI should be robust to impulsive noise. Second, for neighborhoods of sharp transitions in signal the LAI values have to be approximately the same as for the fragments of constant signal.

3. PROPOSED LOCAL ACTIVITY INDICATORS

One disadvantage of the already used quasi-range (QR) and median filtered quasi-range (MQR) LAIs [4] is their insufficient robustness (in cases of rather large probabilities of spike occurrence). Another drawback of QR and MQR is that their values are greatly influenced by sharp signal transitions (for example, the neighborhoods of samples with indices 50, 250, 300 of the test signal shown in Figure). So, now we propose to use two new LAIs: 1) the median of the absolute deviation from the median (MAD) [5], and 2) the so-called "minimized" quasi-range (MQR) specially designed by us.

The MAD parameter for the running window position i is defined as $MAD_i = \mathrm{med}\{|x_j - \mathrm{med}\{x_i\}|\}$, where $\{x_i\}$ is the data sample formed from the values within the current running window and $j = \overline{i - N/2, i + N/2}$. The MQR parameter is expressed as $MQR_i^m = \min_l (Q_l^m)$, $l = \overline{m+1, N-m}$, where $Q_l^m = x^{(l+m)} - x^{(l-m)}$, and m is the parameter that corresponds to the half of the inter-quantile distance $(q-p)/2$ in the usual quasi-range Q_i^{pq}.

The MAD parameter is widely used as robust local scale estimator [5] and, as known, it is more robust than quasi-range. Beside, MAD is less sensitive to the influence of signal sharp transitions.

The use of the minimization operation in the proposed procedure of the MQR allows increasing robustness of the standard QR in the case of identical sign spikes. In such cases QR can reject only p-1 spikes while MQR can reject $2m = q - p = N - 2p + 1$ spikes. Since, for the same considered window size N=9 and for p=3 and q=N-p+1=7 QR can reject only two spikes while MQR can reject four. Because of the same reason, this modification has an improved insensitiveness of LAI to sharp signal transitions.

All the LAI parameters are calculated for the same scanning window position and size N as the proposed AFM output is derived.

4. CONSIDERED SIGNAL/NOISE MODEL AND FILTER PERFORMANCE CRITERIA

As has been mentioned above, we suppose that noise is mixed, and it contains additive and impulsive components. Furthermore, we consider the most complicated case of spikes with identical sign. So the model of a 1-D sampled data sequence we used is

$$y(i) = \begin{cases} S(i) + n_a(i), & \textit{with probability } 1 - P_{imp} \\ S(i) + n_{imp}(i), & \textit{with probability } P_{imp} \end{cases}, \quad (1)$$

where $S(i)$ denotes the true signal value of the i-th sample; $n_a(i)$ is the zero mean Gaussian additive noise with the variance σ_a^2; $n_{imp}(i)$ defines the amplitude of impulsive noise that occurs with the probability P_{imp}. The test signal shown in Figure consists of different fragments peculated to the most of real-live situations: 1) a constant signal (for example, the fragment with indices from 5 to 45); 2) a step edge (positioned on the 50-th sample); 3) piecewise linear curves (indices 90-110 and 190-210); 4) linearly increasing and decreasing signals (for example, the fragments with indices 110-140 and 160-190), 5) a peak-like maximum (90-110); 6) a polynomial maximum (265-285), and 7) piecewise linear and polynomial curves (the junction points have the indices 250 and 300).

As the general characteristic of the AMFs and others concerned filters efficiency we used the MSE criterion χ_t calculated for the entire test signal. Besides, in order to analyze noise suppression efficiency and transitions preservation separately we calculate some local MSE values χ_{loc} for different signal regions (subindices of χ_{loc} in Table 2 correspond to the fragment margins).

5. AMF PARAMETER SELECTION

In order to determinate the k_{SC} value, the special analysis has been performed. For the constant level signal fragment corrupted by only additive Gaussian noise with several variance values σ_a^2=0.01; 0.03; 0.05; 0.1 (σ_a=0.1; 0.17; 0.22; 0.32) the mean values of the LAI parameters calculated for entire signal were obtained. As it has been established, these mean MAD and MQR LAI values relate to σ_a as $\sigma_a \approx 1.5\,\overline{MAD}$ (see also [5]) and

$\sigma_a \approx 1.22\overline{MQR}$, i.e. k_{SC} =1.5 for MAD and 1.22 for MQR.

Generalizing the previously obtained results concerning myriad filter parameter selection [3,4] one can conclude that the optimal $(k/\sigma_a)_{opt}$ value is within the range 0.7÷1.0. So, the optimal value of k_{LAI} is expected to be within the range 1.05÷1.5 for MAD and 0.85÷1.22 for MQR, respectively.

6. NUMERICAL SIMULATION RESULTS

Besides the new AMF versions based on MAD (*Adapt III*) and MQR with m=2 (*Adapt IV*) we also simulated two old AMF versions [4] based on QR with p=3 and q=7 (*Adapt I*) and MQR (*Adapt II*). For comparison we obtained numerical simulation results for the standard median (*Median*) and α-trimmed (*α-Trim*) filters. These results for four different noise environments (different parameters in model (1) - *Cases A, B, C, and D*) are presented in Table 2. The best results (minimal χ values) for each AMF for different k_{LAI} are marked by **bold** formatting and the best result among all considered AMFs is marked by ***bold italic*** formatting.

As seen from Table, for each AMF the optimal value of the k_{LAI} coefficient that provides the best general characteristic χ_t of the filter can be found. The increasing of k_{LAI} value with respect to the value k_{LAI}^{opt} brings to noise suppression improving but signal transition preservation becomes worse. On the contrary, the decreasing of k_{LAI} value leads to noise suppression degradation but signal transition preservation improves.

The best integral characteristics χ_t for all *Cases* are obtained for *Adapt IV* filter (for optimal k_{LAI}^{opt}). As the analysis of the local MSE values shows, in *Cases A*, *B*, and *C*, when the level of the additive and impulsive noise is relatively small, it happens only due to perfect preservation of piecewise linear and polynomial curve (indices 240-260) but for the rest test signal fragments the best results are obtained for *Adapt II* filter. As for the *Case D*, when the destructive influence of mixed noise is very strong, the *Adapt IV* filter is the best for almost all test signal fragments, i.e. this filter is the most robust among the considered AMFs.

The *Adapt III* filter occupies the intermediate place between *Adapt II* and *Adapt IV* filters. This means that MSE χ_t for *Adapt III* filter is usually a little bit worse than for *Adapt IV*, but it is better than for *Adapt II*. On the other hand, almost all χ_{loc} values (except $\chi_{240-260}$) for the *Adapt III* filter are better than for *Adapt IV*, but they are worse than for *Adapt II*.

It is especially interesting that the proposed AMFs provide better preservation of step edge than the standard median filter commonly considered the best in this sense.

CONCLUSIONS

The proposed AMFs based on MAD and MQR LAIs possess good noise suppression efficiency and, at the same time, accurate signal transitions preservation. For the considered test signal and noise properties they improve input SNR by 7...11 dB. These filters (in the sense of total MSE χ_t) outperform the median and α-trimmed filters that are widely used for 1-D signal processing. Moreover, the characteristics of the proposed AMFs are better than for previously designed AMFs, especially for the case of very intensive additive and impulsive noise (σ_a^2=0.03, P_{imp}=0.1).

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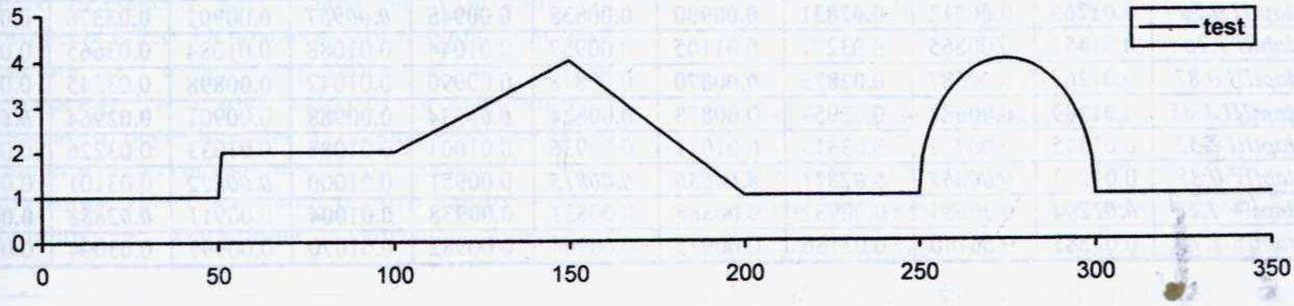

The considered noise-free test signal

Table. The entire and local MSEs for the considered filters (N=9).

	χ_t	χ_{10-40}	χ_{40-60}	χ_{90-110}	$\chi_{110-140}$	$\chi_{140-160}$	$\chi_{160-190}$	$\chi_{190-210}$	$\chi_{240-260}$	$\chi_{265-285}$
Case A: σ_a^2=0.01; P_{imp}=0; n_{imp}=0										
Noise	0.01000	0.00995	0.01016	0.01000	0.01005	0.00998	0.01008	0.00981	0.00991	0.01001
Median	0.00273	0.00166	0.00407	0.00213	0.00237	0.00369	0.00309	0.00244	0.00542	0.00228
α-Trim	0.00441	0.00129	0.02206	0.00153	0.00145	0.00256	0.00154	0.00163	0.01485	0.00160
AdaptI 0.50	0.00316	0.00166	0.00795	0.00212	0.00232	0.00358	0.00291	0.00235	0.00723	0.00223
AdaptII 0.70	**0.00316**	0.00166	0.00795	0.00212	0.00232	0.00358	0.00291	0.00235	**0.00723**	0.00223
AdaptII 1.26	0.00333	***0.00124***	**0.00288**	***0.00150***	***0.00146***	***0.00254***	***0.00147***	***0.00159***	0.01581	***0.00156***
AdaptIII 0.87	0.00290	0.00182	**0.00258**	0.00236	0.00272	0.00406	0.00357	0.00270	0.00581	0.00251
AdaptIII 1.45	**0.00229**	0.00152	0.00434	0.00186	0.00197	0.00314	0.00230	0.00203	**0.00392**	0.00197
AdaptIII 2.32	0.00266	**0.00133**	0.00929	**0.00157**	**0.00154**	**0.00264**	**0.00162**	**0.00169**	0.00657	**0.00167**
AdaptIV 0.85	0.00256	0.00171	***0.00246***	0.00217	0.00243	0.00371	0.00307	0.00244	0.00462	0.00229
AdaptIV 1.28	***0.00218***	0.00148	0.00361	0.00181	0.00190	0.00305	0.00218	0.00197	***0.00374***	0.00191
AdaptIV 1.70	0.00227	**0.00136**	0.00573	**0.00162**	**0.00161**	**0.00273**	**0.00174**	**0.00175**	0.00483	**0.00172**
Case B: σ_a^2=0.01; P_{imp}=0.03; n_{imp}=1.0										
Noise	0.03993	0.03949	0.03783	0.04029	0.03934	0.04253	0.04102	0.03964	0.04015	0.04072
Median	0.00413	0.00185	0.00971	0.00244	0.00271	0.00399	0.00385	0.00304	0.01232	0.00235
α-Trim	0.00531	0.00159	0.02439	0.00192	0.00189	0.00293	0.00232	0.00231	0.01846	0.00179
AdaptI 0.50	0.00423	0.00178	0.01212	0.00222	0.00247	0.00375	0.00328	0.00268	0.01310	0.00226
AdaptII 0.70	**0.00388**	0.00159	***0.00835***	0.00203	0.00226	0.00341	0.00279	0.00243	**0.01373**	0.00201
AdaptII 1.26	0.00443	***0.00142***	0.00962	***0.00169***	***0.00171***	***0.00280***	***0.00204***	***0.00207***	0.01989	***0.00169***
AdaptIII 0.87	0.00412	0.00189	**0.00919**	0.00244	0.00280	0.00415	0.00384	0.00296	0.01200	0.00249
AdaptIII 1.45	**0.00334**	0.00160	0.01034	0.00197	0.00211	0.00332	0.00269	0.00236	**0.00873**	0.00197
AdaptIII 2.32	0.00359	**0.00145**	0.01381	**0.00176**	**0.00180**	**0.00295**	**0.00228**	**0.00215**	0.01068	**0.00173**
AdaptIV 0.85	0.00372	0.00176	**0.00915**	0.00223	0.00251	0.00380	0.00333	0.00269	0.01043	0.00226
AdaptIV 1.28	***0.00322***	0.00155	0.00987	0.00192	0.00205	0.00324	0.00260	0.00231	***0.00838***	0.00191
AdaptIV 1.70	0.00326	**0.00146**	0.01134	**0.00180**	**0.00186**	**0.00302**	**0.00236**	**0.00219**	0.00899	**0.00176**
Case C: σ_a^2=0.03; P_{imp}=0.03; n_{imp}=1.0										
Noise	0.06012	0.06009	0.06176	0.06121	0.06079	0.06154	0.05925	0.06031	0.05879	0.05974
Median	0.00844	0.00559	0.01628	0.00602	0.00638	0.00755	0.00724	0.00673	0.01945	0.00601
α-Trim	0.00852	0.00447	0.02839	0.00481	0.00481	0.00574	0.00508	0.00521	0.02332	0.00467
AdaptI 0.50	0.00817	0.00535	0.01754	0.00564	0.00605	0.00734	0.00680	0.00635	0.01854	0.00583
AdaptII 0.70	**0.00758**	0.00475	***0.01379***	0.00512	0.00553	0.00682	0.00627	0.00574	**0.01947**	0.00527
AdaptII 1.26	0.00776	***0.00424***	0.01850	***0.00453***	***0.00463***	***0.00569***	***0.00496***	***0.00495***	0.02346	***0.00456***
AdaptIII 0.87	0.00852	0.00577	**0.01542**	0.00612	0.00667	0.00806	0.00762	0.00692	0.01893	0.00635
AdaptIII 1.45	**0.00755**	0.00489	0.01890	0.00516	0.00545	0.00664	0.00597	0.00575	**0.01655**	0.00529
AdaptIII 2.32	0.00780	**0.00447**	0.02361	**0.00475**	**0.00487**	**0.00590**	**0.00520**	**0.00523**	0.01974	**0.00477**
AdaptIV 0.85	0.00795	0.00538	**0.01554**	0.00568	0.00615	0.00746	0.00691	0.00639	0.01729	0.00589
AdaptIV 1.28	***0.00736***	0.00478	0.01826	0.00505	0.00533	0.00649	0.00582	0.00562	***0.01608***	0.00518
AdaptIV 1.70	0.00745	**0.00452**	0.02111	**0.00480**	**0.00497**	**0.00603**	**0.00534**	**0.00531**	0.01760	**0.00485**
Case D: σ_a^2=0.03; P_{imp}=0.1; n_{imp}=1.0										
Noise	0.13052	0.12980	0.13029	0.13455	0.13083	0.13012	0.12653	0.13243	0.12454	0.12877
Median	0.01398	0.00784	0.02967	0.01007	0.00978	0.01053	0.01183	0.01063	0.03580	0.00848
α-Trim	0.01513	0.00926	0.03775	0.01168	0.01048	0.01061	0.01128	0.01153	0.03551	0.00893
AdaptI 0.50	0.01266	0.00711	0.02872	0.00891	0.00852	0.00966	0.01010	0.00909	0.03274	0.00778
AdaptII 0.70	**0.01267**	**0.00712**	**0.02831**	**0.00900**	**0.00838**	**0.00945**	**0.00987**	**0.00901**	**0.03376**	**0.00754**
AdaptII 1.26	0.01451	0.00865	0.03271	0.01105	0.00987	0.01044	0.01088	0.01084	0.03665	0.00858
AdaptIII 0.87	0.01267	0.00687	**0.02825**	**0.00870**	0.00848	0.00990	0.01042	**0.00898**	0.03345	0.00793
AdaptIII 1.45	**0.01209**	**0.00681**	0.02959	0.00878	**0.00824**	***0.00934***	**0.00988**	0.00901	**0.02964**	***0.00740***
AdaptIII 2.32	0.01345	0.00772	0.03313	0.01011	0.00936	0.01003	0.01088	0.01033	0.03226	0.00805
AdaptIV 0.85	0.01213	***0.00667***	***0.02811***	***0.00850***	***0.00815***	0.00951	0.01000	***0.00872***	0.03101	0.00759
AdaptIV 1.28	***0.01204***	0.00683	0.02937	0.00888	0.00831	**0.00938**	**0.01004**	0.00911	***0.02888***	**0.00742**
AdaptIV 1.70	0.01283	0.00740	0.03136	0.00975	0.00904	0.00982	0.01070	0.00997	0.03004	0.00781