THE COMBINED SYSTEMS OF SUPERVISION

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Abstract. The algorithm allowing optimum is offered to restore object under its discrete images generated by several linear systems. The cases are given, when its application allows to receive considerably accurate(up to tenfolds) results of restoration in comparison with results of restoration of the image generated by one, even best, system.

1. INTRODUCTION

The numerical processing of the images with the purpose of reception of the substantial information about the optical characteristics of observable object is usually connected to the solutionof the Fredholm integrated equation of the first kind. In practice, the object can be observed by several not ideal systems. However analyze is going only with results of supervision by the best system and, hence, by solving the equation, appropriate to it. But the systems of supervision can be nonideal on any other business and even the "bad" systems can contain such information on object, which lacks in the best ones. Therefore, we believe, that would be useful to develop the algorithm of supervision of the same object by several systems and to restore the information on object optimum by results of supervision by these systems. Let's name such systems of supervision of objects as the combined systems.

2. Optimum algorithm of restoration of objects under the images generated by combined systems.

The process of formation of the data by the combined system is simulated by the following equation

$$\int_{-\vec{S}}^{\vec{S}} z(\vec{\xi})K(\vec{x},\vec{\xi},p)d\vec{\xi} = f(\vec{x},p) + \gamma(\vec{x},p) = F(\vec{x},p), \quad |\vec{x}| \le \vec{R}, \tag{1}$$

Where $z(\vec{\xi})$ are the required characteristics of object,

$$\vec{\xi} = (\xi_1, \xi_2, \dots, \xi_M),\ d\vec{\xi} = d\xi_1 d\xi_2 \dots d\xi_M$$
$$\vec{S} = (S_1, S_2, \dots, S_M),\ \vec{x} = (x_1, x_2, \dots, x_N),$$
$$d\vec{x} = dx_1 dx_2 \dots dx_N,\ \ \vec{R} = (R_1, R_2, \dots R_N).$$

The inequalities of a type $|\vec{x}| \le \vec{R}$ designate $|x_i| \le R_i$ for everyone i. Integer variable p designates number of separate system. The right part of expression (1) is known approximately as $f(\vec{x},p)$ -is exact value of the right part, $\gamma(\vec{x},p)$ is error of its given value (noise). Square-integrated kernel $K(\vec{x},\vec{\xi},p)$ of the equation (1) we shall define (determine) by the following expression:

$$K(\vec{x},\vec{\xi},p) = \begin{cases} A_1 K(\vec{x},\vec{\xi},1) \\ A_2 K(\vec{x},\vec{\xi},2) \\ \dots\dots\dots\dots\dots \\ A_P K(\vec{x},\vec{\xi},P) \end{cases} \tag{2}$$

Here - A_p, $p = 1,2,\dots,P$ normalised (in case of the different registrators they have- different dimensions) factors chosen so that dispersion of noise of all systems of supervision was identical. As algorithm of data processing registered by the combined systems is chosen stabilized [1] algorithm of restoration of objects, as it at any discretisation of the initial data is optimum with a rating of accuracy in mean-squared metrics and is applicable even if the equation (1) has no single-valued solution . The kind of it practically does not vary for the combined systems with any nucleuses such as (2). For the explanatory told we shall give it completely.
The required object is represented as an expansion () on any system of basic functions

$$z(\vec{\xi}) = \sum_{k=1}^{\infty} c_k \psi_k(\vec{\xi}), \quad |\vec{\xi}| \le \vec{S}. \tag{3}$$

Then, the "approached" decision of the equation (1) is represented as

$$z(\vec{\xi},\vec{\beta}) = \sum_{k=1}^{\infty} c_k(\vec{\beta})\psi_k(\vec{\xi}) = \tag{4}$$

$$= \sum_{m=1}^{\infty} \frac{\sum_{l=1}^{m} d_{lm}(\vec{\beta})\psi_l(\vec{\xi}) \sum_{k=1}^{m} d_{km}(\vec{\beta})(\varphi_k, F)}{\beta_m + \sum_{k=1}^{m} d_{km}(\vec{\beta})(\varphi_k, \varphi_m)}.$$

Here $\varphi_k(\vec{x},p) = \int_{-S}^{S} \psi_k(\vec{\xi})K(\vec{x},\vec{\xi},p)d\vec{\xi}$

are the images of basic functions, and

$$(\varphi_k,\varphi_l)=\sum_{x,p}\varphi_k(\vec{x},p)\varphi_l(\vec{x},p)$$

are their scalar products. (The summation will be carried out(spent) on all values discretised for input in the computer of arguments of the image). The "approached" values of factors of decomposition (3) are calculated under the formulae

$$c_l(\vec{\beta})=$$

$$=\sum_{m=l}^{\infty}d_{lm}(\vec{\beta})\frac{\sum_{k=1}^{m}d_{km}(\vec{\beta})(\varphi_k,F)}{\beta_m+\sum_{k=1}^{m}d_{km}(\vec{\beta})(\varphi_k,\varphi_m)}.$$

The factors $d_{ln}(\vec{\beta})$ are calculated on by the recurrent formula

$$d_{ln}(\vec{\beta})=$$

$$=-\sum_{m=l}^{n-1}d_{km}(\vec{\beta})\frac{\sum_{k=1}^{m}d_{km}(\vec{\beta})(\varphi_k,\varphi_n)}{\beta_m+\sum_{k=1}^{m}d_{km}(\vec{\beta})(\varphi_k,\varphi_m)},$$

$$l=1,2,\ldots,n-1;\ n=1,2,\ldots.$$

Thus $d_{kk}(\vec{\beta})=1,\ k=1,2,\ldots,$ and if $l>n$, then $d_{ln}(\vec{\beta})=0$.

The "approached" solution (4) depends on stabilizing vector parameter $\vec{\beta}=(\beta_1,\beta_2,\ldots)$.

. Its optimum value is calculated from a condition of a minimum mean-squared error

$$\rho^2=\left\langle\int_{-\bar{S}}^{\bar{S}}\left[z(\vec{\xi},\vec{\beta})-z(\vec{\xi})\right]^2d\vec{\xi}\right\rangle$$

At statistical conditions:

à) $\langle c_k c_i\rangle=\langle c_k^2\rangle\delta_{ik}$, á) $\langle c_k\gamma_i\rangle=0$,

â) $\langle\gamma_i\gamma_k\rangle=\gamma_*^2\delta_{ik}$,

(uncorrelated object, object and noise are uncorrelated, noise is uncorrelated). Here γ_*^2 -is dispersion of an error with zero average at definition of values $f(\vec{x},\vec{\xi},p)$ for input in the computer, the angular brackets mean averaging on set of realizations. Optimum values of stabilizing parameter we shall mark by an asterisk. Their values: $\beta_m^*=\gamma_*^2/\langle c_m^2\rangle$ .. At such values of stabilizing parameter minimal mean-squared values of an error are as Kind

$$\langle\Delta c_l^2(\vec{\beta}^*,\gamma_*)\rangle=\left\langle\left[c_l(\vec{\beta}^*,\gamma_*)-c_l\right]^2\right\rangle =$$

$$=\sum_{m=1}^{\infty}\frac{d_{lm}^2(\vec{\beta}^*)\langle c_m^2\rangle\gamma_*^2}{\gamma_*^2+\langle c_m^2\rangle\sum_{k=1}^{m}d_{km}(\vec{\beta}^*)(\varphi_k,\varphi_m)} \qquad (5)$$

$$\rho^2(\vec{\beta}^*,\gamma_*^2) =$$

$$=\sum_{m=1}^{\infty}\frac{\langle c_m^2\rangle\gamma_*^2\sum_{l=1}^{m}d_{lm}^2(\vec{\beta}^*)}{\gamma_*^2+\langle c_m^2\rangle\sum_{k=1}^{m}d_{km}(\vec{\beta}^*)(\varphi_k,\varphi_m)}. \qquad (6)$$

Last two formulas allow to estimate increase of accuracy of definition of object at use of the combined system in comparison with results of restoration of the image generated by any one individual system. So, for example, if as results of measurement by the combined system to consider(examine) two independent images of object, blurred at the expense of rectilinear uniform movement during an exposition on mutually perpendicular to directions, the account under the formulas (5), (6) shows increase of accuracy in tenfolds (at small γ_*^2). This fact is caused by that in the image, blurred on a direction x of value of a spatial spectrum of object on frequencies $(0,\omega_y)$ remain undistorted. In the second image, on the contrary, keep the values of a spectrum of object on frequencies $(\omega_x,0)$.. Thus, at use of optimum algorithm of restoration, the mistakes are sharply reduced at joint numerical processing of two images. Other example gives a roentgen tomography, which too can be considered(examined) as the combined system, on one projection to receive 3-dimentional distribution because it is impossiblewith a help of one projection results. .

3. CONCLUSION

The results of work show, what not always it is necessary to ignore "bad" systems of supervision. Their sharing with usual ones can essentially improve accuracy of restoration of an object, so and raise probability of its recognition.

REFERENCES

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