Increasing sensitivity and space resolving power of low contrast radiological image analysis as a task inverse analytical continuation of virtual pseudocoherent wave field

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Abstract **- Objective: Increase in the resolving power and sensitivity of low-contract images analysis. Method: It is based on representation of an original image as a complex envelope of some hypothetical pseudocoherent wave field. The imaginary part of the new complex image is connected to its real part by means of the 2-D Hilbert transform. It is supposed that the complex envelope satisfy conditions for wave equation in the original image registration plane. The effect of controlled levels of the contrast and space resolving power is ensured using the wave equation inversion procedure in depth with further analysis of the series of new virtual images. Result: Very good results were obtained while processing many CT and MR low-contrast images and space born ones.**

The problem of low contrast image (LCI) enhancement and segmentation is one of the most important for medical diagnostics and remote sensing.

Very often the task of a LCI enhancement can be represented as an one of inverse filtering within the framework of the equation of convolution [1].

$$i(x,y)_{obs} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} H(x-u, y-v)\, i(u,v)_{id}\, du\, dv, \qquad (1)$$

where observed image $i(x,y)_{obs}$ being considered as a result of the influence of some distortion operator $H(x,y)$ onto an unknown ideal image $i(x,y)_{id}$.

In this case a problem of the LCI enhancement can be formulated as that of inverse filtering.

The inaccuracy of distortion of the operator H can be reduced to the influence of a structural and measurement noise $n(x,y)$. Then the inverse filtering problem is reduced to a search of a corresponding regularized solution $\hat{i}(x,y)_{id}$ [2].

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$$\hat{i}(x,y)_{id} = \frac{1}{(2\pi)^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{I(k_x,k_y)}{H(k_x,k_y)} e^{-j(k_x x + k_y y)} \times \frac{dk_x dk_y}{1+\alpha(k^2{}_x + k^2{}_y)e^{-\beta(k^2{}_x+k^2{}_y)^{1/2}}} \qquad (2)$$

where $I(k_x,k_y)$ and $H(k_x,k_y)$ are results of the Fourier transform of $i(x,y)_{obs}$ and $H(x,y)$ accordingly; k_x and k_y – space frequencies. Parameter β in (2) depends on behavior of the ratio $I(k_x,k_y)/H(k_x,k_y)$ when $(k_x^2+k_y^2)^{1/2}$.

Regularization parameter β must be coordinated with the accuracy of the image $i(x,y)_{obs}$ registration.

There are many publications (the comprehensive bibliography is given in [3]) devoted to consideration of various approaches intended for solving the equation (1), but, however, under the condition that analytical description of the distortion operator H in some form is known. However, in application to tomographic LCI (especially for MRI), the principal problem is the fact that the form of $i(x,y)_{obs}$ depends on too many unknown physical and technical factors [4]. The latter means that it is practically impossible to consider the problem of increasing the quality of the tomographic LCI $i(x,y)_{obs}$ (as example) on base of equation (2), because the analytical description of the distortion operator $H(x,y)$ is unknown. This very fact conditions availability of a great deal of the purely mathematical algorithms of the secondary processing of LCI [5].

In the paper, the information possibilities of the new LCI processing method are considered. The presented approach is based on the physical inversion method which does not require any *a priory* analytical description of $H(x,y)$ and providers effective simultaneous segmentation of LCI via using of new phase-space characteristics. The method is based on some assumptions:

Assumption 1. The original LCI $i(x,y)_{obs}$ is the real part of a complex envelope $s(x,y)$ of an unobserved coherent wave field $g = g(x,y,z,t)$ which spreads up along the axis z from $z=-\infty$ to $z=0$:

$$g(x,y,z,t) = s(x,y)\exp[j(k_z z - \omega t)] \quad (3)$$
$$k_z = 2\pi/\lambda; \qquad \omega = 2\pi f;$$
$$i(x,y) = \mathrm{Re}\{s(x,y)\},$$

where f is the frequency and λ -- the wavelength of the hypothetical wave field. In this sense $g = g(x,y,z,t)$ can be considered as a pseudocoherent (virtual) wave field (PWF). It is supposed that complex envelope $s(x,y)$ of the PWF be "frozen" in the plane $z=0$ of the observation of the original image $i(x,y)_{obs}$ at the moment $t=0$, i. e.

$$i(x,y)_{obs} = i(x,y) = \mathrm{Re}\{s(x,y,z=0,t=0)\}, \quad (4)$$

Assumption 2. The imaginary part of the complex envelope of the PWF $q(x,y) = \mathrm{Im}\{s(x,y)\}$ is connected to its real part $i(x,y) = \mathrm{Re}\{s(x,y)\}$ by means of the 2-D Hilbert transform

$$q(x,y) = \frac{1}{\pi^2} P \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{i(u,v)}{(u-x)(v-y)} du dv, \quad (5)$$

where P stands for the Cauchy principal value. This allows to express the complex envelope of the PWF in the form

$$s(x,y) = i(x,y) + jq(x,y) = a(x,y)\exp(j\varphi(x,y)) \quad (6)$$
$$a(x,y) = \sqrt{i^2(x,y) + q^2(x,y)},$$
$$\varphi(x,y) = arctg[q(x,y)/i(x,y)], \quad (7)$$

where $a(x,y)$ and $\varphi(x,y)$ denote the magnitude-space and phase-space characteristics, respectively.

The Fourier transform of the $i(x,y)_{obs} = i(x,y)$

$$I(k_x, k_y) = F\{i(x,y)\} =$$
$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} i(x,y)\exp[-j(k_x x + k_y y)] dx dy \quad (8)$$

is connected to the complex envelope $s(x,y) = s(x,y,z=0,t=0)$ of the PWF by the expression

$$s(x,y) = 2\int_0^{\infty}\int_0^{\infty} I(k_x,k_y)\exp[j(k_x x + k_y y)] dk_x dk_y \quad (9)$$

In equation (8), k_x and k_y denote space frequencies, F – the operator of the Fourier transform.
The complex envelope characteristics $a(x,y)$, $\varphi(x,y)$, $\mathrm{Re}\{s(x,y)\}$ and $\mathrm{Im}\{s(x,y)\}$ are considered as the sought-for information in the framework of the proposed method.

Remark. On this stage is possible to realize an algorithm for providing effect of a virtual lens using with varying focus depth. As it is known, the complex envelope of a coherent optical wave on the output of the lens with transfer function $T(x,y)$ has form [6]

$$S_{out}(x,y) = T(x,y)S(x,y), \quad (10)$$

where

$$T(x,y) = C\exp(-jk\rho^2/2f_0);$$
$$C = \exp(jk\eta d) \quad (11)$$

In (11) $k = 2\pi/\lambda$; d – maximum thickness of the virtual lens; η - virtual coefficient of refraction; f_0 – focus depth of the lens; $\rho = (x^2+y^2)^{1/2}$. From equation (11) is seen that virtual lens $T(x,y)$ providers only phase correction of the input complex envelope $S(x,y)$.

From principle point of view, a digital variation of the focus depth f_0 (as controlled parameter) allows enhance the quality of a virtual analysis of the LCI, but there is one practical difficulty. The $S(x,y)$ is given in the rectangular coordinate system, but $T(x,y)$ is computed in the polar co-ordinate system. The influence of a pixel structure of the $S(x,y)$ will be displayed in appearance of artifacts (there is some analogy with the slice theorem in tomography). Experiments had shown that ones influence on the phase-space characteristics of the $S(x,y)_{out}$ was very significant. For this reason the other method of the phase-correction was used.

Assumption 3. PWF $g(x,y,z,t)$ satisfies the wave equation

$$\frac{\partial^2 g}{\partial t^2} = c^2\left(\frac{\partial^2 g}{\partial x^2} + \frac{\partial^2 g}{\partial y^2} + \frac{\partial^2 g}{\partial z^2}\right) \quad (12)$$

with boundary conditions for PWF complex envelope $\mathrm{Re}\{s(x,y,z=0,t=0)\} = i(x,y)_{obs}$. In (12) c stands for the phase velocity of the PWF and in general case $c = c(x,y)$.

Method:. The solution to the problem of increasing the LCI contrast and spatial resolving power is inversion of the wave equation (12) in depth from plane of the original image registration $z=0$ to $z=z_i$, $i=1,2,...,N$, i.e. the problem is to restore a complex wave field $g(x,y,z)$ in the time moment $t=0$ from the wave field $g(x,y,z=0,t=0)$ which exists in the plane of original observation.

There are various possible approaches to the problem of the wave equation (12) inversion. The used method's outline description in application to the idealized situation $c = const$ is as follows. Using the properties of the Fourier transform, from the equation (12) we obtain

$$F\left\{\frac{\partial^2 g}{\partial t^2}\right\} = (j\omega)^2 G(k_x,k_y,k_z,\omega),$$

$$F\left\{\frac{\partial^2 g}{\partial y^2}\right\} = (jk_y)^2 G(k_x,k_y,k_z,\omega), \qquad (13)$$

$$F\left\{\frac{\partial^2 g}{\partial x^2}\right\} = (jk_x)^2 G(k_x,k_y,k_z,\omega),$$

$$F\left\{\frac{\partial^2 g}{\partial z^2}\right\} = (jk_z)^2 G(k_x,k_y,k_z,\omega).$$

Taking the Fourier transform of both parts of the equation (12) along co-ordinates x, y and t, the following expression can be obtained:

$$\frac{d^2G(k_x,k_y,z,\omega)}{dz^2} = \left(\frac{\omega^2}{c^2} - k_x{}^2 - k_y{}^2\right) G(k_x,k_y,z,\omega) \qquad (14)$$

Equation (14) is simple differential equation of second order, which has two independent solutions corresponding to waves of two types. For our problem only the solution for the downward extrapolation from $z = 0$ to $z = z_i$ is of interest. The solution of the extrapolated PWF has the form

$$G(k_x,k_y,z+\Delta z,\omega) = \exp\left(j\Delta z\sqrt{\frac{\omega^2}{c^2} - k_x{}^2 - k_y{}^2}\right) G(k_x,k_y,z,\omega) \qquad (15)$$

or

$$G(z+\Delta z) = KG(z),$$

where

$$K = \exp\left(j\Delta z\sqrt{\frac{\omega^2}{c^2} - k_x{}^2 - k_y{}^2}\right), \quad |K| = 1. \qquad (16)$$

1) From equation (16), it is seen that PWF extrapolation is based on the PWF front phase correction.

From dependence $G(k_x,k_y,z,\omega)$, one can obtain needed solution $g(x,y,z,t=0)$ applying the 2-D Fourier inverse transform in the plane of space frequencies k_x and k_y with subsequent summation in time frequencies ω [7]:

$$g(x,y,z,t) = \sum_{k_x}\sum_{k_y}\sum_{\omega} G(k_x,k_y,z,\omega) \times \exp[j(k_x x + k_y y)] \qquad (17)$$

From equations (15) and (16) it is seen that specific solution depends on the choice of the PWF virtual wavelength, depth of the PWF extrapolation and spatial dependence of the PWF phase velocity $c = c(x,y)$.

Result.

Example of the X-ray CT image of a coxa joint is shown in fig.1.

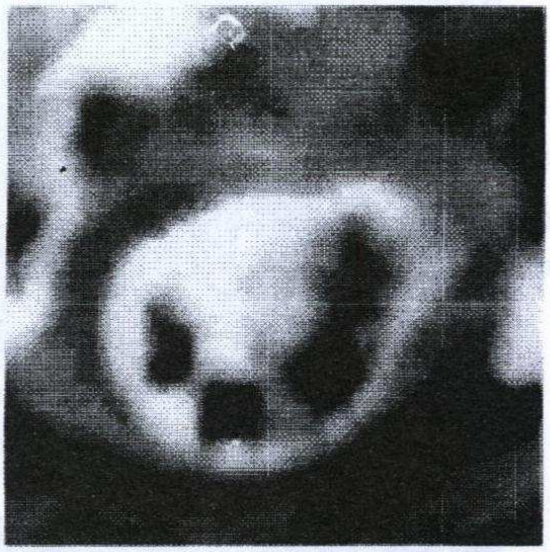

Fig 1. Example of low contrast biomedical image X-ray CT

Very important practical feature of the PWF inversion method is the possibility of aiming at the most interesting LCI domains for their detailed analysis.

CONCLUSION

1. The experimental results confirmed the practical expedience of using the PWF inversion method for low-contrast image processing.
2. The PWF inversion method ensures high spatial resolving power and high-sensitive segmentation for low-contrast image analysis.
3. Very important practical feature of the method lies in the possibility of the aiming at the most interesting domains of the low-contrast images by means of proper choice of the PWF extrapolation depth. From physical point of view, such an effect is equivalent to introduction of the variable focus depth for the new virtual images.

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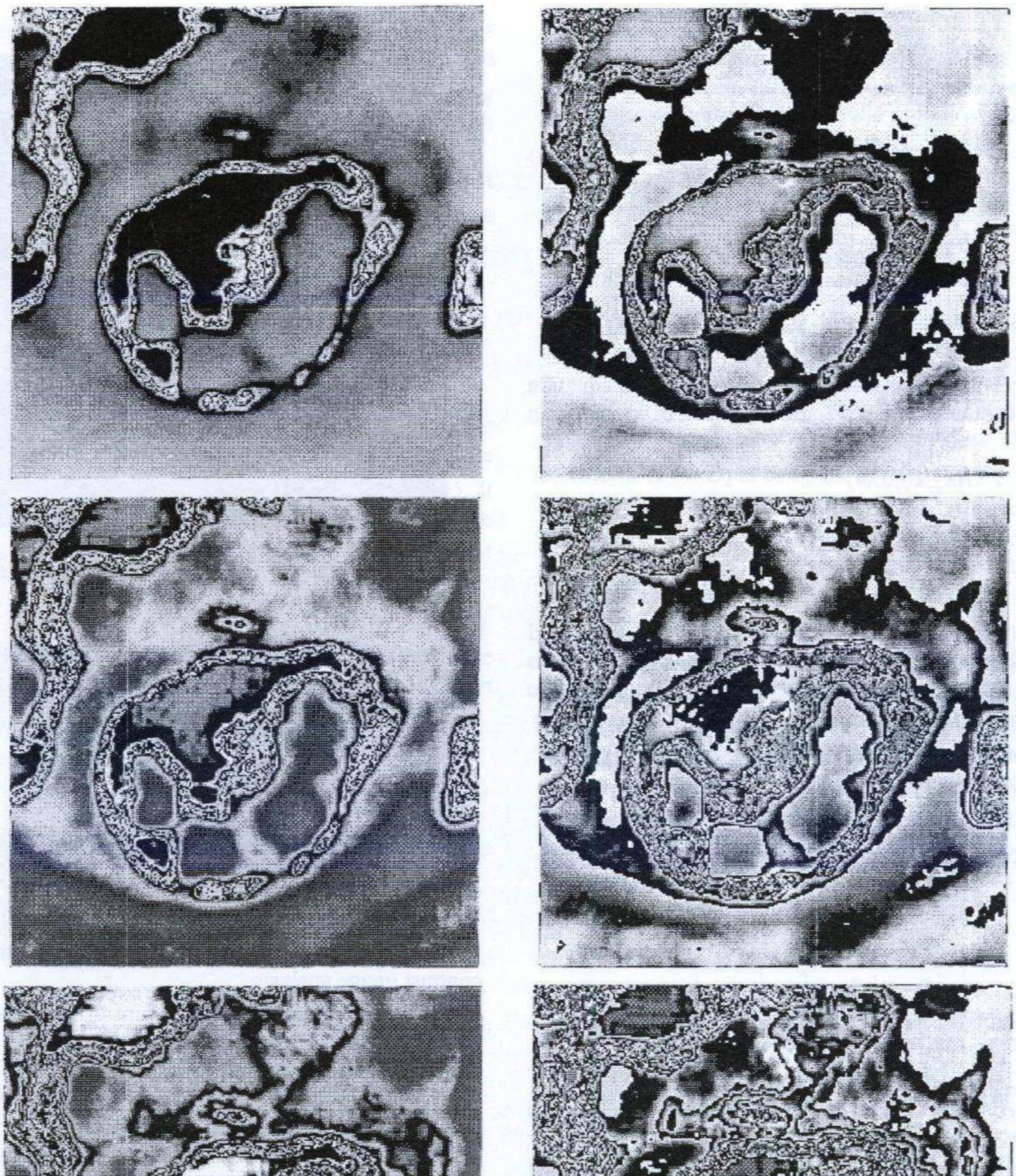

Fig.2 The complex envelope S(x,y) for various values of z_i of the extrapolated PWF for the CT image: 1 – a(x,y); 2 - φ(x,y).