Nonlinear model of qualitative classification on base of wave-packet decomposition in space of Volterra kernels.

Akhmetshin A.M., Shutin D.V.

Dnepropetrovsk State University
Email: akhm@mail.dsu.dp.ua, faria@email.dp.ua

ABSTRACT

In the given paper the new approach to the usage of nonlinear models for qualitative classification has been presented. The method is oriented on classification of texture images and stochastic signals. The wave packet decompositions of the initial sequence were interpreted as input and output signals of virtual nonlinear system. The peculiarities of nonlinear transfer characteristics were than considered for performing a qualitative classification. This technique was applied to classification of gastritis on the base of IR spectrograms of the mucous membrane, x-ray CT and x-ray images of the bone trabecular structure, affected with osteoporosis. It was shown, that the proposed method requires much smaller resolution and it is also more sensitive to the changes in the initial sequence.

Keywords: Nonlinear identification, Volterra kernels, wave-packet decomposition.

1. INTRODUCTION

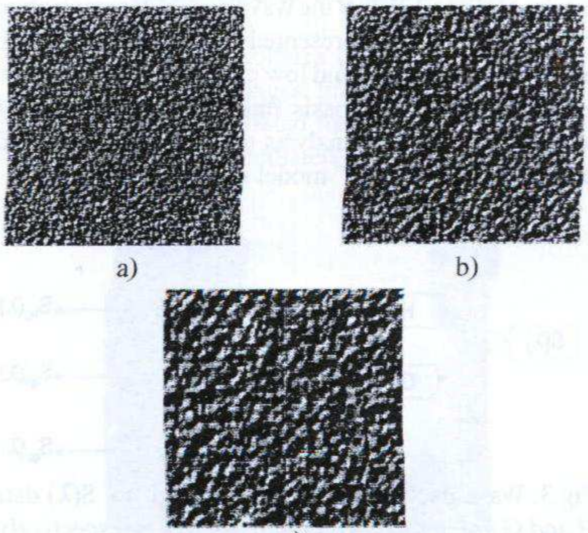
Stochastic processes (signals) find their application in almost every natural system. This fact very often makes them an object of research. In terms of information theory, signal acquired from the system can be interpreted as a message, transmitted to the receiver. The receiver's main role is to "read" the message correctly, in other words, to identify some distinctive characteristics of the message (signal) and classify it. The correct classification of the information received plays one of the most important roles in further processing. In case of medical data processing we would most often speak of correct diagnostics. Therefore, choosing the proper method of signal classification would increase the probity of the results received and quality of the analysis as a whole.

Current methods of data classification are based on linear and nonlinear models. Linear models are most often simple enough. They can be easily simulated and analyzed, but these models are quite rough and unsuitable for the complex processes. On the other hand, the usage of nonlinear models lacks simplicity, but it does provide an exactness and accurateness of the results obtained.

In the given paper we presented a method of qualitative classification of the stochastic signals and images, which is based on nonlinear models and wave-packet decomposition.

2. METHOD

Let's consider three grained texture images (Fig.1.). To increase the sensitivity of the processing procedure, it was proposed to analyze the texture image by the cut. The cut should preserve the spectral information of the image however it will lose information of any objects that are on the texture.



a) b) c)

Fig.1. Three grained texures.

The taken cuts $S(\lambda)$ of the presented images are shown on Fig.2. In case of stochastic signals, it would be convenient to use AR models[1] for describing them, i.e.

$$S(\lambda) = \sum_{i=1}^{M} P_i S(\lambda_{k-i}) + a(\lambda_{k-i}) \text{ ; k=1,...,N.} \quad (1)$$

In terms of the model (1), observable data $S(\lambda)$ can be interpreted as an output of a linear system excited with a stationary white noise $a(\lambda_k)$. This approach is very efficient when the processes, being simulated, are stationary. However in many cases these signals are non-stationary. It means that the input of our system must be nonstationary as well. In this case we should use another approach to specify the input of the system so that the requirements mentioned above were satisfied. To achieve this, the wave packet decomposition[2] was used. It was

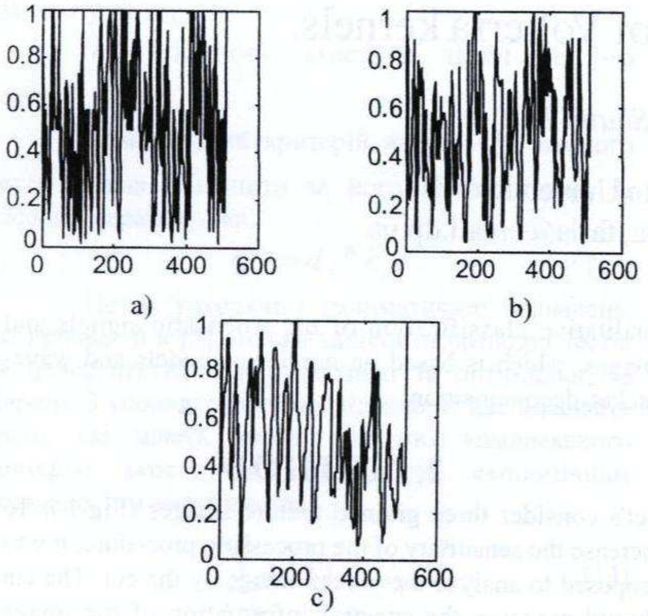


Fig.2. Slices of the grained textures, presented on the Fig.1.

shown by Mallat S.[2], that the wave packet decomposition (till the 2 level)could be presented as it is shown on Fig.3. *H* and *G* are special high and low pass filters respectively. They play a role of the basis functions in wave-packet decomposition. Further analysis of S(λ) was conducted in terms of "input-output" model shown on Fig.4.

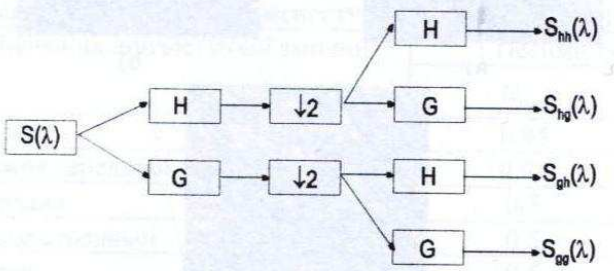


Fig.3. Wave packet decomposition of given S(λ) data; H and G are lowpass and highpass filters respectively; ↓2 – dyadic downsampling.

It was assumed, that $S_{hh}(\lambda)$ and $S_{gh}(\lambda)$ are the input and output of the model respectively. Practicability of this approach has following grounds:

1. $S_{hh}(\lambda)$ and $S_{gh}(\lambda)$ are sharing the same "genetic root" (i.e. S(λ)) and have concerted Fourier spectra. It ensues from the idea of wave packet decomposition.
2. $S_{hh}(\lambda)$ and $S_{gh}(\lambda)$ are non-stationary alternating-sign sequences, which envelope curves are correlated. This fact is the main reason for using model, presented in Fig.4.

The main idea of the analysis is the connection between $S_{hh}(\lambda)$ and $S_{gh}(\lambda)$ through functional second order Volterra series. As long as the second order Volterra kernel $F(\tau_1,\tau_2)$ is an image it should reflect the changes in the signal, making them more obvious for a visual observation. This fact specified the choice of the second order Volterra kernel as a major informative characteristic for the qualitative classification. The kernels of the system with arbitrary input and output sequences can be calculated in the frequency domain by non-parametric method of Powers[3].

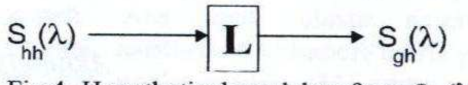


Fig.4. Hypothetical model to form $S_{gh}(\lambda)$; L - nonlinear operator, characterizing the system excited with $S_{hh}(\lambda)$.

Volterra kernels, calculated for the grained textures(Fig.1) are shown on Fig.5.

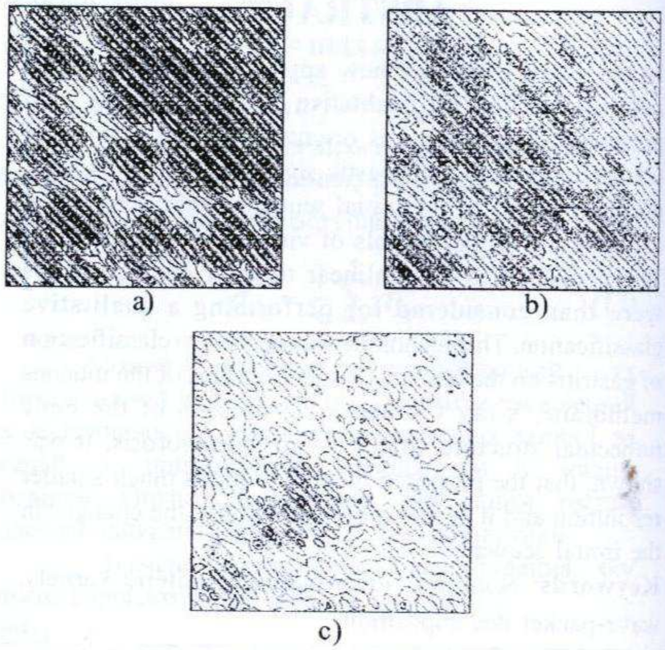
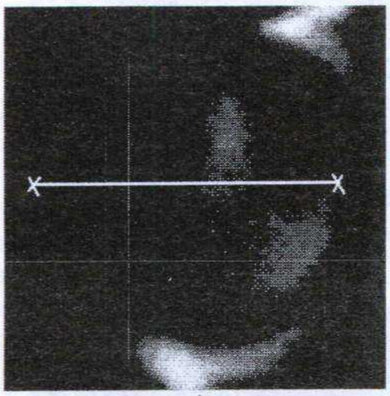


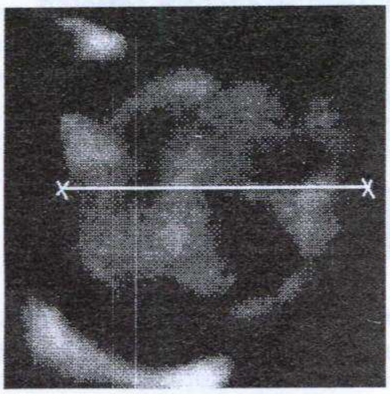
Fig.5. Volterra kernels, calculated for textures, shown on the Fig.1.

3. APPLICATION OF THE PROPOSED METHOD

The proposed method has been applied to qualitative characterization of different stochastic processes, in particular to characterization X-Ray CT and X-Ray images of a coxa, affected with osteoporosis and IR spectrograms of gastric mucos membrane. X-Ray CT images of coxa are presented on Fig.6. The main advantage of using X-Ray CT images is that they give the full information of the bone topology, revealing the local changes of trabeculae. This fact makes the diagnostics of osteoporosis on the base of the tomographic images very effective, especially, on the beginning stages of the disease development. X-Ray CT images, shown on Fig.6. present coxae of a woman at the age of 45. The right coxa (b) is affected more deeply with osteoporosis than the left one (a). This fact has been proved by earlier clinical analysis. White lines show the places where the cuts have been taken from the image for the analysis. It should be noted, that only the oscillating component was used for the analysis because it is responsible for the trabecular bone structure. Therefore, only this component should be analyzed for osteoporosis diagnostics. The Volterra kernels, calculated by the proposed method form these component are shown on Fig.7. Considering calculated kernels, it can be seen that the peculiarities of the kernel topology reflect the changes in bone trabecular structure and therefore they can be used for osteoporosis diagnostics. The proposed method was also applied to the diagnostics of osteoporosis on the base of X-Ray images. One of the classical methods for osteoporosis classification on the base of X-Ray images is an X-Ray densitometry techniques, which are based on dual energy x-ray absorptiometry[4]. This approach gives good results, but the resulting images could not be processed any further, because the brightness of the pixels is connected to the density of the bone and any nonlinear transformation would destroy this brightness/density relationship. Therefore, the method, which could be applied to the X-Ray images acquired with a usual x-ray camera, would be of a great use.On Fig.8. there are two X-Ray images of the left coxa of a person suffering from a quickly developing osteoporosis.The treatment began in 1997 but the drug therapy failed to stop the disease development and in two years the coxa was replaced with prosthesis. It was assumed that a cut of the X-Ray image (the projection of trabecular structure on a plain surface) should contain averaged sizes of trabeculae. Therefore, local changes in trabeculae bone structure should be reflected in local characteristics of oscillating component of a slice. From looking at calculated images of second order Volterra

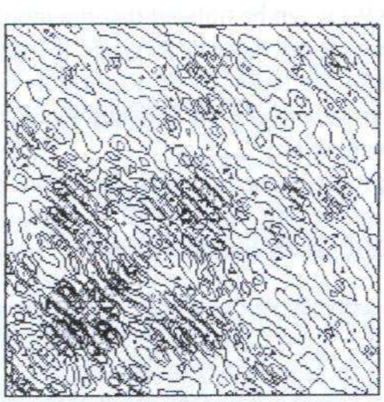


a)

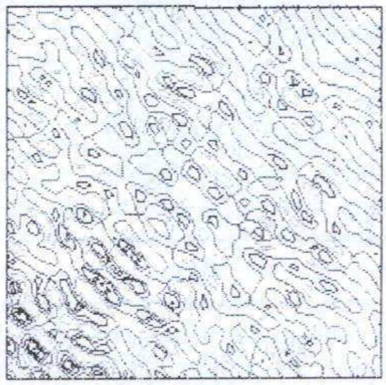


b)

Fig.6. X-Ray CT images of the human coxae, affected with osteoporosis.
a) normal coxa; b) affected with the desease.

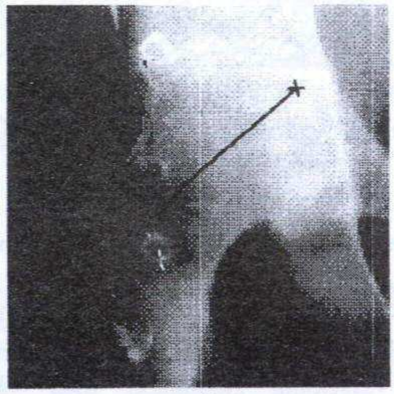


a)

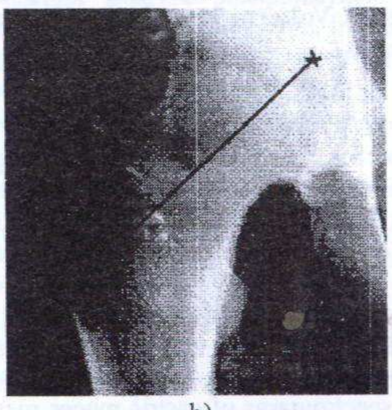


b)

Fig.7. Second order Volterra kernels, calculated for the X-Ray CT images, presented on Fig.5.
a) kernel calculated for the normal coxa;
b) kernel calculated for the ill coxa.



a)

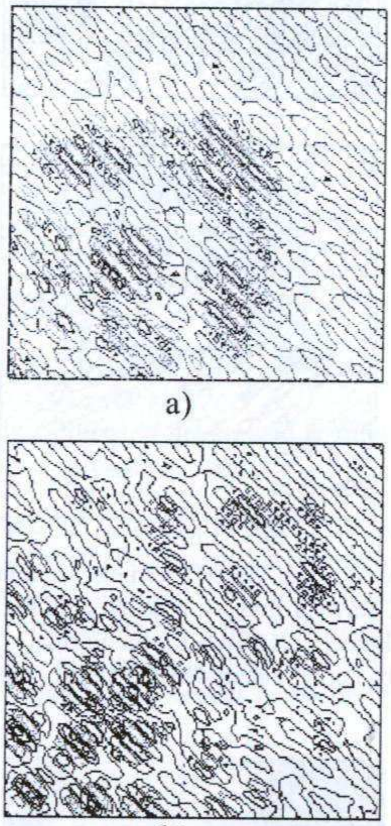


b)

Fig.8. X-Ray images of the coxa,affected with osteoporosis. a) Year 1998;b) Year 1999.

kernels (Fig.9.) it can be noticed that the topology of the image changes as the disease develops. To our mind, the peculiarities of the topology of the kernels cohere with the trabecular structure – the less uniform kernel is, the thicker trabeculae are.

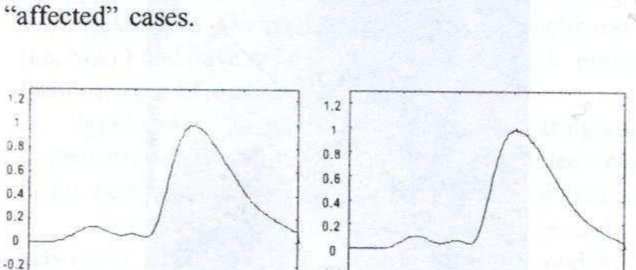
The proposed method was also tested on gastritis classification on the base of IR spectroscopy data (Fig.10.) of the gastric mucos membrane. In comparison


a)

b)

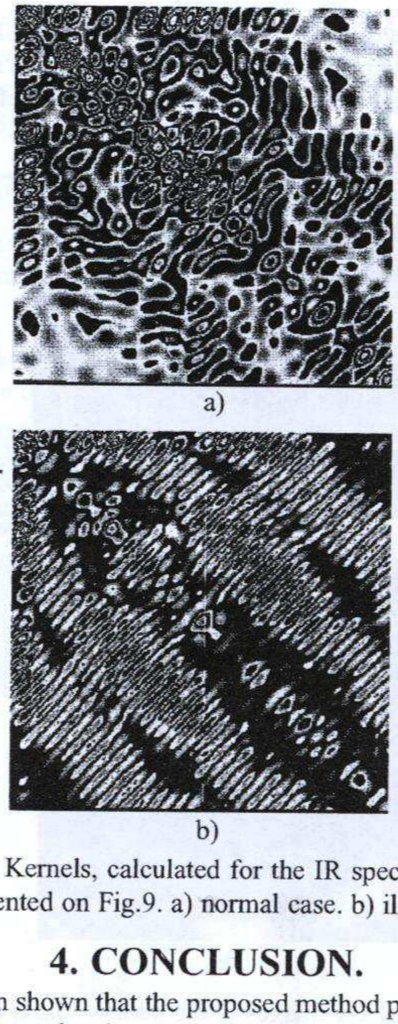
Fig.9. Kernels, calculated for the X-Ray images. presented on Fig.7.
a) Year 1998; b) Year 1999

with other methods of testing for gastroenterological diseases, IR spectroscopy is the most harmless and easy to be conducted. The use of the proposed method gives very good results, especially, when the calculated kernels were further processed (Fig.11.)[5]. Contrasting images on Fig.11 gives us an opportunity to perform qualitative distinctions of Volterra kernels for "normal" and "affected" cases.



a) b)

Fig.10. IR spectrograms of gastric mucos membrane; a) normal; b) gastritis;


a)

b)

Fig.10. Kernels, calculated for the IR spectrograms presented on Fig.9. a) normal case. b) ill case.

4. CONCLUSION.

It has been shown that the proposed method proved to be sensitive enough to be successfully applied to the analysis of trabecular bone structure on the base of both X-Ray CT and X-Ray images. The results of IR spectroscopy data analysis have also proved that this method can be effectively used for gastritis diagnostics. Nonlinear approach and wave-packet decomposition, implemented in the method, make it effective in analysis of complex stochastic processes, especially, when the physico-mathematical model of the processes being studied is very complex.

REFERENCES.

[1]. Box G.E.P., Jenkins G.M., *Time series forecasting and control*, San Francisco, 1970.

[2]. Mallat S., "A theory for multidimensional signal decomposition: the wavelet representation", IEEE Trans. On Pattern Analysis, v.11, pp.674-693, 1989.

[3]. Powers, E.J., "Applications of digital polyspectral analysis to nonlinear systems modeling and nonlinear wave phenomena," *Proc. Workshop on Higher-Order Spectral Analysis*, pp. 73-77, Vail, Colorado, June 1989.

[4]. Lafferty F.W. "Correlation of dual-energy X-ray absorptiometry, quantitative computed tomography, and single photon absorptiometry with spinal and non-spinal fractures". *Osteoporosis Int* 6:407-415, 1996.

[5]. Akhmetshin A., "Pseudocoherent phase-contrast imaging method with varying focus depth: an effective tool of low-contrast radiological image analysis", SPIE Proc., Medical Image Processing, v.3338, pp.904-915, 1998.