MYRIAD FILTER PROPERTIES AND PARAMETER SELECTION

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ABSTRACT

Peculiar properties of myriad filters in case of their application to 1-D signal processing in mixed noise environment are considered. It is shown that the filter output bias and the filter noise suppression effectiveness considerably depend upon filter parameter *k*, signal behavior and noise characteristics. Some recommendations concerning parameter *k* selection are given.

1. INTRODUCTION

In processing the signals corrupted by mixed, i.e. additive and impulsive, noise the nonlinear filters have shown themselves to be a useful and efficient tool since they are able simultaneously to remove spikes, suppress additive noise component and preserve signal abrupt changes well enough [1], [2].

A new and recently introduced class of nonlinear data processing algorithms is a myriad filter and its modifications like weighted myriad filters [3], [4]. They have been proposed and designed as nonlinear filters for robust non-gaussian signal processing in impulsive noise environment, in particular, in case of α-stable distributions of noise having heavier tails than gaussian and, for some cases, Laplacian ones.

The performance characteristics of the standard myriad filter strongly depend on the so-called linearity parameter k that controls the outlier-rejection capability of an estimator. At the same time, this parameter determines the dynamic (output bias) and statistical characteristics of the myriad filter. Certainly, all these characteristics also depend upon a scanning window size.

Some of these properties have not been exploited in [3], [4] and for their analysis we apply approaches described in our papers [5], [6]. It is demonstrated, in particular, that myriad filters with rather small *k* (just under this condition an appropriate robustness is provided) distort piece-wise fragments of signal in peculiar manner and even "amplify" noise for rapidly increasing/decreasing signals. These drawbacks are undesirable and should be avoided. To get around this problem is possible due to proper, compromise, selection of parameter *k*. In order to perform it we present below the computer simulation results and give the corresponding recommendations based on test signal analysis.

2. MYRIAD FILTER DEFINITION

A sample myriad is an M-estimator of location. Given a set of samples $\{y(i)\}, i = 1,\ldots,N$, an M-estimator of location is described by expression $\hat{\theta} \overset{\Delta}{=} \arg\min_{\theta} \sum_{i=1}^{N} \rho(y(i)-\theta)$, where $\rho(.)$ is the cost function. The sample myriad is defined using the cost function $\rho(y) = \log(k^2 + y^2)$. Therefore, for standard myriad filter output the used cost function is $\sum_{i=1}^{N} \log(k^2 + (y(i)-\theta)^2)$ where $\{y(i)\}$ is formed by sample values within the scanning window of size N. In other words, the output $x(j)$ of the myriad filter applied to process $\{y(j)\}, j = 1,\ldots,J, J >> N$ is written as follows

$$x(j) = \arg\min_{\theta} \sum_{i=j-(N-1)/2}^{j+(N-1)2} \log(k^2 + (y(i)-\theta)^2). \quad (1)$$

Another, sometimes more convenient, way of the myriad filter output definition is expressed as

$$x(j) = \arg\min_{\theta} \prod_{i=j-(N-1)/2}^{j+(N-1)/2} (1 + (y(i)-\theta)^2 / k^2). \quad (2)$$

It can be easily shown that the sample myriad reduces to the sample mean as $k \to \infty$. At the same time, as $k \to 0$ the sample myriad tends to the distribution mode and this is a rather important property. That is why we used earlier the terms "rather small" and "rather large" values of parameter k. However, one should keep in mind that these relative definitions depend upon signal scale.

Other details concerning the myriad and weighted myriad filter peculiarities can be found in [3], [4]. Our purpose is to attract the reader's attention to specific aspects that could be important in practical applications.

3. CONSIDERED SIGNAL/NOISE MODEL

Let us consider the following signal/noise model of a 1-D sampled data sequence

$$y(j)=\begin{cases} S(j)+n_a(j), \text{ with probability } 1-P_{imp} \\ S(j)+n_{imp}(j), \text{with probability } P_{imp}, \end{cases}$$

(3)

where $S(j)$ denotes the true signal value of the j-th sample; $n_a(j)$ is the zero mean gaussian additive noise with the variance σ_a^2; $n_{imp}(j)$ defines the amplitude of impulsive noise that occurs with the probability P_{imp} and it is supposed that $|n_{imp}| > \sigma_a$.

The considered test signal is presented in Fig.1, **test**. The behavior of the myriad filter output was studied for different fragments of this signal: 1) a constant signal (for example, the fragment with indices from 10 to 40); 2) a step edge (indices 40-60); 3) piecewise linear curves (indices 90-110 and 190-210); 4) linearly increasing and decreasing signals (110-140 and 160-190); 5) peak maximum (140-160); 6) polynomial maximum (265-285); 7) a specific fragment (240-260). In other words, this set of test fragments covers a rather wide variety of situations taking place in practice. Thus, the analysis performed in this paper characterizes the myriad filter output peculiarities from quite different points of view.

4. OUTPUT BIAS

Myriad filter mean output considerably depends upon noise characteristics even if the noise is gaussian. This property is, in general, typical for many nonlinear filters [1], [2], [5]. However, since for rather large k the myriad filter characteristics approximate the standard mean filter ones, it is not worth considering the myriad filter bias for this case. The parameter k selection to be rather small when essentially nonlinear and robust properties of the myriad filter are provided is of more theoretical and practical interest.

The studies done have shown that in case of small k, considerably less than the step edge height Δh, the myriad filter preserves it better than a standard median filter with the same N. This conclusion is valid for wide range of additive noise variances $\sigma_a << \Delta h$.

In general, the properties of all nonlinear filters also depend upon scanning window size N. Below we analyze only the case N=9, however, the character of main dependences is similar for other N within the limits from 5 to 11.

However, very peculiar effects take place in the neighborhoods of indices 100, 200, 250, 300, i.e. for piecewise curve junction points. These effects are illustrated in Fig. 2. When k is rather small (0.01) the myriad filter output bias derived as $\Delta_{myr}(j)=E[x(j)]-S(j)$ can have sign that is opposite to the output bias sign in case of large k. For instance, for the neighborhood of the sample with index 100 the myriad filter output bias is positive but very small for j<100, and for j=101, 102, and 103 it becomes negative (if k=0.01). Finally, with further j increasing the myriad filter mean $E[x(j)]$ quickly "jumps" and becomes approximately equal to $S(j)$ (See Fig. 2,a). Similar effects are observed for k=0.01 in the neighborhoods of sample indices 200, 250 (See Fig.2,b), and 300. At the same time, for k=1.0 the myriad filter output behavior is much more typical and it is quite similar to many other nonlinear (for example, α-trimmed) and linear filters [5] (Consider curves for k=1.0 in Fig.2,a and 2,b). All these effects are explained by the mode selection property of myriad filters with small k.

The Table 1 presents the numerical simulation results for the test signal and different noise characteristics. The values of mean square errors (MSE) have been evaluated for the entire signal (χ_t) and locally for its fragments. For example, χ_{10-40} is the local MSE evaluated for the signal fragment from the index 10 to the index 40 and the MSE χ_{40-60} just corresponds to the step edge neighborhood. The MSEs do not characterize the output bias directly since they also take into account the variance of noise remained after filtering. However, the contribution of output bias into local MSEs is considerable in step edge and junction point neighborhoods. That is why the MSE values give reliable information for analysis of filter output bias for many fragments except constant, linearly increasing/decreasing signal (where bias is zero for case of spike absence) and polynomial maximum fragments. In general, polynomial maximum is preserved by myriad filter well enough if k is not rather large.

The dynamic errors (output bias) in the neighborhood of peak extremum (index 150) depend upon k and usually they slightly increase with parameter k increasing. It should be also mentioned that except of output bias the considerable growth of residual noise variance can be observed in the neighborhoods of junction points (indices 100, 150, 200, 250, 300) if k is rather small.

And this is one more drawback of the myriad filters. As it can be seen from Table 1 for *Case A* (small variance of gaussian noise and spikes are absent), the smallest MSE values in the step edge neighborhood are provided by the myriad filter with k=0.01. At the same time for other junction point neighborhoods the better results are observed for myriad filters with k=1.0 and k=10.0. This means that a trade-off between noise suppression efficiency and detail preservation ability of the myriad filter should be found and it is reachable by means of parameter k selection (variation).

5. NOISE SUPPRESSION EFFICIENCY AND ROBUST PROPERTIES

In case of rather small k the peculiar effects can be observed for constant and linearly increasing/decreasing signal (LIDS) fragments. The noise suppression efficiency of nonlinear filter can be assumed sufficient if it does not differ a lot from the corresponding efficiency of the standard mean filter with the same N [6]. For example, for the best nonlinear noise suppressing filters like Wilcoxon and Hodges-Lehman ones the residual noise variance is not larger than by 30% in comparison to mean filter for LIDS [6]. The myriad filter with rather small k satisfies this requirement for constant signal (analyze χ_{10-40} in Table 1 for *Case A*). But note that in this situation the value of k is comparable with σ_a. If the ratio σ_a / k increases the difference between noise suppression efficiency of the myriad and mean filters for constant signal radically becomes larger. This means that the parameter k selection can be scale dependent and for constant signal the sample scale is determined by noise standard deviation σ_a (if the spikes are absent).

Another tendency is observed for LIDSs. Results in Table 1 (consider $\chi_{110-140}$ and $\chi_{160-190}$ for *Case A*) show that the variance of "residual" noise for k=0.01 is larger than even σ_a^2. That is the myriad filter with rather small k can "amplify" noise. And this is a very serious drawback to be avoided. Now let us take into account that the sample scale in this situation is mainly determined not by noise standard deviation but by parameter $\Delta S_{sc} = N\Delta S$ where $\Delta S = |S(j+1) - S(j)|$. Thorough studies have demonstrated that by selecting $k > 0.3\Delta S_{sc}$ it is possible to appropriate noise suppression efficiency of the myriad filter for LIDS. And the selection k=0.01 does not satisfy this inequality for LIDS of the considered test signals. This is also the reason why large residual noise variances are observed for k=0.01 in the neighborhoods of samples with indices 150 (peak maximum), 250 and 300 (junction points of polynomial maximum with constant signals). Therefore, the conclusion is that for almost all the signal fragments except step edge neighborhood the proper selection of parameter k can be done on basis of local estimation of sample scale. But one should keep in mind that k increasing leads to worse robustness with respect to spikes and also results in larger output bias.

To confirm the fact that the myriad filter robustness sufficiently depends upon k let us briefly analyze the numerical simulation data for *Case B* (spike presence) in Table 1. As seen, the myriad filter with k=0.01 out performs the myriad filters with k=1.0 and 10.0 for some signal fragments in the sense of smaller local MSE. In particular, this takes place for constant signal fragment, step edge neighborhood and even the fragment with indices from 90 to 110 (first piecewise curve junction neighborhood). The main reason are the good robust properties of the myriad filter with k=0.01. However, the best, i.e. the smallest MSEs χ_t for entire signal are provided by the myriad filter with k=1.0 for both *Case A* and *Case B*. These interesting results serve as a motivation for locally adaptive approach application to signal processing on basis of myriad filters with varied parameters. Several approaches to providing such filter robustness are worth exploring. One of them is the use of robust estimators of scale.

CONCLUSIONS

The definition of the myriad filter is given and its advantages and drawbacks are considered. The basic characteristics of the myriad filter can be tuned by selecting the only parameter k but it should be a compromise decision depending upon the required trade-off between filter robustness, noise suppression efficiency and output bias. Numerical simulation results and performed analysis results give a starting point for further studies.

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Table 1.The MSEs for the considered filters

	χ_{I}	χ_{10-40}	χ_{40-60}	χ_{90-110}	$\chi_{110-140}$	$\chi_{140-160}$	$\chi_{160-190}$	$\chi_{190-210}$	$\chi_{240-260}$	$\chi_{265-285}$
Case A: σ_a^2=0.001; P_{imp}=0.00, n_{imp}=0.00										
Noisy	0.00099	0.00102	0.00100	0.00097	0.00098	0.00094	0.00101	0.00095	0.00101	0.00098
K=0.01	0.00742	0.00013	0.00015	0.00058	0.00116	0.00244	0.00276	0.00152	0.05298	0.00076
K=1.0	0.00357	0.00011	0.02462	0.00030	0.00011	0.00122	0.00011	0.00051	0.01457	0.00042
K=10.0	0.00631	0.00011	0.03525	0.00029	0.00011	0.00125	0.00011	0.00051	0.03135	0.00043
Case B: σ_a^2=0.01; P_{imp}=0.03 n_{imp}=1.00										
Noisy	0.03961	0.04662	0.03825	0.04058	0.03804	0.03711	0.04083	0.03457	0.03641	0.03998
K=0.01	0.01097	0.00218	0.01291	0.00316	0.00435	0.00637	0.00773	0.00506	0.05543	0.00365
K=1.0	0.00670	0.00274	0.02879	0.00332	0.00288	0.00345	0.00308	0.00373	0.02041	0.00257
K=10.0	0.01134	0.00515	0.04034	0.00541	0.00501	0.00551	0.00508	0.00596	0.03738	0.00455

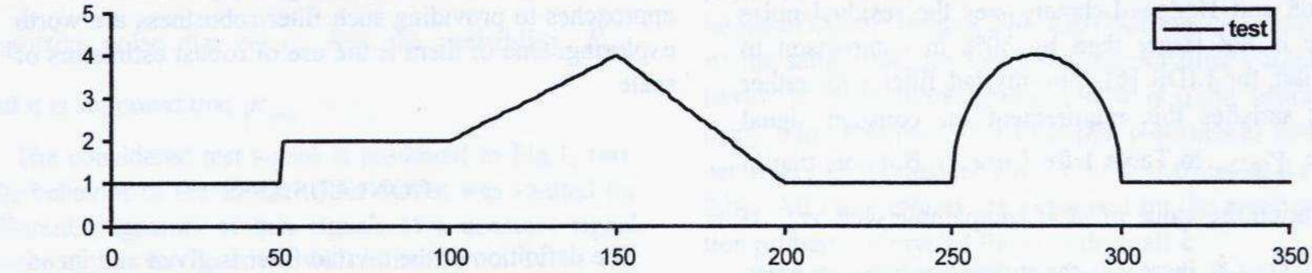
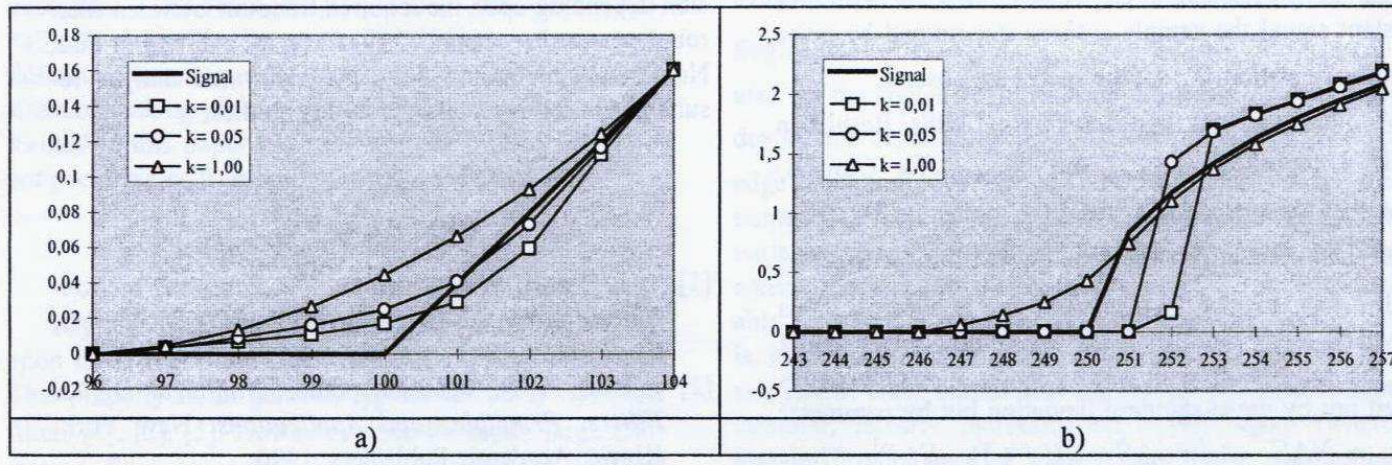


Figure 1. The considered noise-free test signal.



Figures 2. Myriad filter mean outputs for piecewise curve fragments (only gaussian noise, *k*=0.01, 0.05 and 1.0): a) neighborhood of sample 100; b) neighborhood of sample 250.