

CONFLICTFREE PARALLEL MEMORY ACCESS MODELS FOR IMAGE PROCESSING

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Abstract. There are essential peculiarities when images of different nature are parallel processed. On numerous occasions subframe processing occupies a special role in filtering, restoration, correction etc. To decrease computing time, an approach to synthesis of memory structures is proposed. Single-cycle access is ensured for simultaneous processing of both all pixels belonging to arbitrary subframe and ensemble of pixels which are chosen per one from several overlapping subframes. Page scans are proposed as a toolkit to be exploited to obtain various memory models. All algorithms based on subframe processing can be formulated in page scans terms. Results presented can be easily extended to multidimensional data with parallel processing purpose.

INTRODUCTION

To date parallel and distributed computing (pipelined vector processors, transputers, PRAM (Parallel Random Access Machine), VSA (Virtual Systems Architecture), BSP (Bulk Synchronous Parallel) models etc.) continuously plays increasing role in image processing in general. If algorithms are based on the calculation of local image properties in all possible neighbourhoods of the image, there are essential peculiarities in processing of subframes (windows). From point of view of time overhead often memory access procedures occupy an essential long in the whole computing time. From earliest studies [1-3] the most steadfast attention was given to parallel memories for scanning fields or for data in matrix form. Classes of periodic, isotropic and linear functions were offered as toolkit for synthesis of conflictfree data exchange between the computing environment and memory modules [4-7].

We present point-to-set maps as toolkit for parallel memory structures creation for image processing. The subject in this study is 2-D signals but all got results can be easily extended to N-dimensional cases. Page scans are proposed to obtain memory modules with single-cycle access. Sequential subframe (all pixels of an arbitrary subframe are processed simultaneously) and parallel subframe (anywise pixels are chosen per one from a series of overlapping subframe and they are processed simultaneously) treatments can be formulated in terms of

the proposed page scans.

I. PAGE SCANS AND MEMORY STRUCTURES

Let $A = \{1, 2, \dots, M\} \times \{1, 2, \dots, N\}$ be an address space of an digital image, i. e. elements of this Cartesian product identify displacement of all pixels B_{ij} in videobuffer. Moreover, let introduce subframe Ω as a subset of image address space

$$\Omega = \{i_1, i_2, \dots, i_m\} \times \{j_1, j_2, \dots, j_n\} \subseteq A, \\ i_{s+1} = i_s + 1, j_{r+1} = j_r + 1, \\ s \in \{1, 2, \dots, m-1\}, r \in \{1, 2, \dots, n-1\}.$$

We can analyse rectangular subframe without loss of generality as in the cases of an arbitrary shape $\Gamma \subset \Omega$ we are only to redetermine the function of brightness

$$B_{ij} = \begin{cases} B_{ij}, (i, j) \in \Gamma, \\ 0, (i, j) \in \Omega \setminus \Gamma. \end{cases}$$

It should be emphasised that a choice of subframe linear size substantially depends on task orientation and parallel execution environment.

Let $\Pi(A)$ be the aggregate of all non-empty subsets of A . Among all possible point-to-set maps $F: A \rightarrow \Pi(A)$ we distinguish the system $\{F_\beta\}_{\beta \in D(\beta)}$ ($D(\beta)$ is index set for point-to-set maps choice) satisfying following relations

$$\forall a \in A, F_\beta(a) = P_q \subset A, 0 \leq q \leq \text{card } A - 1; \quad (1)$$

$$\bigcup_{a \in A} F_\beta(a) = A; \quad (2)$$

$$\forall a_1, a_2 \in A : a_1 \neq a_2 \Rightarrow \\ \Rightarrow \begin{cases} F_\beta(a_1) = F_\beta(a_2), \\ F_\beta(a_1) \cap F_\beta(a_2) = \emptyset; \end{cases} \quad (3)$$

$$\forall \Omega \subseteq A, \forall a_1, a_2 \in \Omega : a_1 \neq a_2 \Rightarrow \\ \Rightarrow F_\beta(a_1) \cap F_\beta(a_2) = \emptyset. \quad (4)$$

Let us agree that address set P_q is called memory page and let us agree that point-to-set maps from (1) - (4) are called page scans. The essence of a page scan lies in address relocation, when sampling, so that for given size relation all elements of an arbitrary subframe have to state in different real blocks of storage. Therefore, single-cycle access is attained without increasing of memory capacity.

For instance, let us indicate point-to-set maps as page scans pointed out

$$F_{\beta'}(a_{ij}) = P_q, a_{ij} \in P_q, q =$$

$$= \text{rem} \{ (i + (j - 1)m - 1) / \text{card } \Omega \},$$

$$F_{\beta''}(a_{ij}) = P_q, a_{ij} \in P_q, q =$$

$$= \text{rem} \{ (j + (i - 1)n - 1) / \text{card } \Omega \},$$

where «rem(\odot)» denotes a remainder under division.

To verify that $F_{\beta'}$ is the page scan, it suffices to check property (4) since relations (1) – (3) are obvious in this case. Hence, we have to examine implication

$$\forall a_{ij}, a_{pq} \in \Omega \subseteq A \Rightarrow \begin{cases} F_{\beta'}(a_{ij}) = P_{i'} \\ F_{\beta''}(a_{ij}) = P_{i''} \end{cases} \Rightarrow i' \neq i''$$

where

$$i' = \text{rem} \{ (i + (j - 1)m - 1) \},$$

$$i'' = \text{rem} \{ (p + (q - 1)m - 1) \}.$$

Let us assume the contrary, namely, $i' = i''$. Then there is a divisibility of the value $|i + (j - 1)m + (p - (q - 1)m)|$ by $\text{card } \Omega$ or, that is the same, the following equality holds

$$|(i - p) + m(j - q)| = r \text{card } \Omega, \quad (5)$$

where r is some natural number. In accordance with $\forall a_{ij}, a_{pq} \in \Omega$ we have

$$|i - p| \leq m - 1, \quad |j - q| \leq n - 1. \quad (6)$$

Taking into account (5) and (6) we get

$$|(i - p) + m(j - q)| \leq |i - p| + m|j - q| \leq$$

$$\leq m - 1 + m(n - 1) = mn - 1.$$

At the same time $mn - 1 < \text{card } \Omega$, consequently, $r = 0$ and therefore $|i - p| = m|j - q|$. From inequalities (6) it follows $|m(j - q)| \leq m - 1$ that can be valid only under condition $|j - q| = 0$, i.e. $q = j$ and similarly $p = i$. Thus we draw the conclusion about the contradiction to the initial assumption. As a consequence the point-to-set map $F_{\beta''}$ also generates the page scan.

Notice that according to (4) $\text{card} \{F_{\beta}(a)\}_{a \in A} \geq \text{card } \Omega$, hence we shall say: point-to-set map $F : A \rightarrow \Pi(A)$ induces an optimal page scan if $\text{card} \{F_{\beta}(a)\}_{a \in A} = \text{card } \Omega = mn$, in other words, if number of memory blocks equals number of subframe pixels.

Emphasise that point-to-set map satisfying (1) – (4) determines only the storage page therefore full address word is following diagonal product of maps

$$F_{\beta} \Delta \mathfrak{Z} : A \rightarrow \Pi(A) \times \mathfrak{R}^{\tau}$$

where F_{β} is page scan, \mathfrak{Z} is some map which restriction

$$\mathfrak{Z}|_{F_{\beta}^{-1}(a) \in A} : \{F_{\beta}(a)\}_{a \in A} \rightarrow \mathfrak{R}^{\tau}$$

is injection, i.e.

$$\forall a_1, a_2 \in \{F_{\beta}^{-1}(a)\}_{a \in A}, a_1 \neq a_2 \Rightarrow \mathfrak{Z}(a_1) \neq \mathfrak{Z}(a_2).$$

Introducing of τ -dimensional space into preceding map may be explained by hardware or system software environment specialities, for instance, by a hierarchical page segmentation, an insufficiency of digits per address word for direct access, masking of some pixels, etc.

Let us evaluate total number of all possible optimal page scans. Since property (4) is valid for arbitrary page scan, choose subframe as shown in Fig. 1. It is clear that there exist $mn!$ pixel reallocations for subframe chosen. Under displacement in direction a) for one cell pages $P_{n+1}, P_{n+2}, \dots, P_{2n}, \dots, P_{(m-1)n+1}, P_{(m-1)n+2}, \dots, P_{mn}$ are already fixed and it is necessary to fulfil pixel readdressing to pages P_1, P_2, \dots, P_n . It is obvious that $n!$ variants are possible. But then to satisfy property (4) there is only one variant under displacement in direction b). Judging

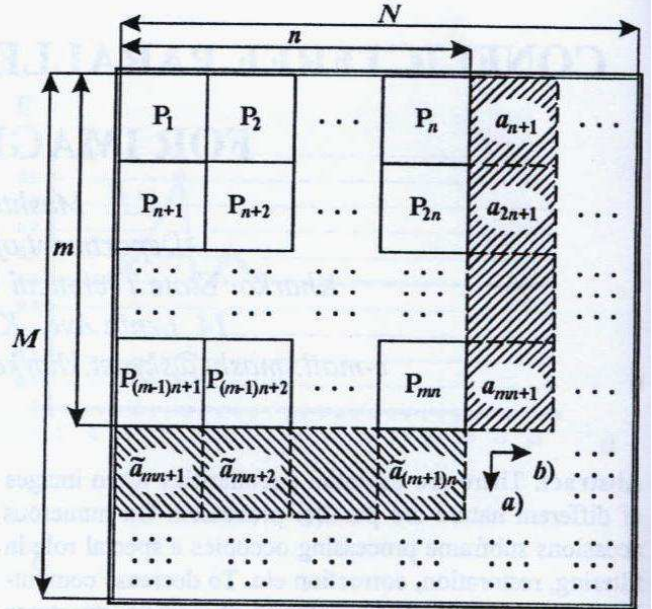


Fig. 1. Page scans number evaluation

similarly we have $m!$ variants of pixel readdressing to pages $P_{n+1}, P_{2n+1}, P_{3n+1}, \dots, P_{(m-1)n+1}$ under initial displacement in direction b) and we have one variant in direction a). Hence, under condition $N \geq 2n, M \geq 2m$, implementing displacement $N - n, M - m$ times again, and repetition of initial subframe readdressing being accounted for, finally we get

$$K = (m!)^{N-n} + (n!)^{M-m} - 1.$$

Considering rest of possible relations between address space and subframes parameters, we summarise, up to $(mn)!$, total number of all possible optimal page scans

$$K = \begin{cases} (m!)^{N-n} + (n!)^{M-m} - 1, & N \geq 2n, M \geq 2m; \\ (m!)^{N-n} [(2n - N)!]^{M-m} + \\ + (n!)^{M-m} [(2m - M)!]^{N-n} - 1, & n \leq N \leq 2n, \\ & m \leq M \leq 2m; \\ (n!)^{M-m} [(2m - M)!]^{N-n} + \\ + (m!)^{N-n} - 1, & N \geq 2n, m \leq M \leq 2m; \\ (m!)^{N-n} [(2n - N)!]^{M-m} + \\ + (n!)^{M-m} - 1, & n \leq N \leq 2n, M \geq 2m. \end{cases}$$

By virtue of what has already been said, we can draw conclusion that among all possible page scans it is desirable to choose only simplified (from point of view amount of address computation) multivalued maps.

II. PRODUCING PAGE SCANS BY ADDRESS INDICATORS

We shall focus attention on regular page scan in connection with that the calculation of pixel addresses should be realized as it is possible easier. Let us define parameter β , selecting page scans $\{F_{\beta}(a)\}_{a \in A}$, as vector-function $f(i, j) \in \mathfrak{R}^{\tau}, (i, j) \in A$. Let function $f(i, j)$ extend readdressing law to all address space under page scan given for arbitrary subframe. Let us agree to say that such function is address indicator and let us define it axiomatically:

i) restriction

$$f|_{\{1, 2, \dots, m\} \times \{1, 2, \dots, n\}} : \{1, 2, \dots, M\} \times \{1, 2, \dots, N\} \rightarrow \mathfrak{R}^{\tau}$$

is injection, i.e.

$$\begin{cases} \forall i_1, i_2 \in \{1, 2, \dots, m\}, \forall j_1, j_2 \in \{1, 2, \dots, n\} : \\ : (i_1, j_1) \neq (i_2, j_2) \Rightarrow f(i_1, j_1) \neq f(i_2, j_2); \end{cases}$$

ii) address indicator has translation invariant relation of equality, i.e.

$$\begin{aligned} \forall (i_1, j_1), (i_2, j_2) \in \{1, 2, \dots, m\} \times \{1, 2, \dots, n\}, \\ \forall (i_1 + \Delta i, j_1 + \Delta j), (i_2 + \Delta i, j_2 + \Delta j) \in A \end{aligned}$$

if $f(i_1, j_1) = f(i_2, j_2)$, then

$$f(i_1 + \Delta i, j_1 + \Delta j) = f(i_2 + \Delta i, j_2 + \Delta j).$$

It is easily to prove that an address indicator $f(i, j)$ induced a page scan $F_\beta(a_{ij})$ determining by $A_{f(i, j)}$. Indeed, consider arbitrary subframe $\Omega \subset A$ with linear sizes $m \times n$. We are to check that from condition $\forall (i, j) \neq (p, q), a_{ij}, a_{pq} \in \Omega$ it follows $f(i, j) \neq f(p, q)$. This inequality corresponds to assertion $F_\beta(a_{ij}) \neq F_\beta(a_{pq})$ (property (4) of page scans).

Let us assume the contrary, namely, $f(i, j) = f(p, q)$. Setting $\Delta i = \min(i, p) - 1, \Delta j = \min(j, q) - 1$ and always supposing $i' = i - \Delta i, p' = p - \Delta i, j' = j - \Delta j, q' = q - \Delta j$, we obtain $1 \leq i' \leq |p - i|, 1 \leq j' \leq |q - j|$. Taking into account that $a_{ij}, a_{pq} \in \Omega$ we have

$$(i', j'), (p', q') \in \{1, 2, \dots, m\} \times \{1, 2, \dots, n\}$$

and from translation invariant relation of equality and the condition $f(i, j) = f(p, q)$ it follows $f(i', j') = f(p', q')$ which in accordance with property i) contradicts the assertion that $f(i, j)$ is an address indicator. Notice that since in this case a page scan is determined by relationship $F_\beta(a_{ij}) = A_{f(i, j)}$ then properties (1) - (3) are obvious. It now becomes clear that to find the regular page scan it suffices to check conditions i), ii).

Now let us consider an example of optimal address indicator (optimalness of address indicators is similar to that of page scan)

$$f(i, j) = n \operatorname{rem} \left\{ \frac{i}{m} \right\} + \operatorname{rem} \left\{ \frac{j-1}{n} \right\}. \quad (7)$$

We first note that condition i) is obvious, indeed

$$\begin{aligned} f(\{1, 2, \dots, m\} \times \{1, 2, \dots, 1n\}) = \\ = \begin{bmatrix} n & n+1 & \dots & 2n-1 \\ 2n & 2n+1 & \dots & 3n-1 \\ \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots \\ (m-1)n & (m-1)n+1 & \dots & mn-1 \\ 0 & 1 & \dots & n-1 \end{bmatrix}. \end{aligned}$$

To complete the check it remains to show the validity of translation invariant relation of equality. Rewrite

$$f(i_1, j_1) = f(i_2, j_2)$$

in the form

$$n \left| \operatorname{rem} \left(\frac{i_1}{m} \right) - \operatorname{rem} \left(\frac{i_2}{m} \right) \right| = \left| \operatorname{rem} \left(\frac{j_1-1}{n} \right) - \operatorname{rem} \left(\frac{j_2-1}{n} \right) \right|.$$

The assumption: $\operatorname{rem} \left(\frac{i_1}{m} \right) - \operatorname{rem} \left(\frac{i_2}{m} \right) \neq 0$ implies that inequalities

$$\begin{aligned} n \left| \operatorname{rem} \left(\frac{i_1}{m} \right) - \operatorname{rem} \left(\frac{i_2}{m} \right) \right| &\geq n, \\ \left| \operatorname{rem} \left(\frac{j_1-1}{n} \right) - \operatorname{rem} \left(\frac{j_2-1}{n} \right) \right| &\leq n-1 \end{aligned}$$

hold which contradicts $f(i_1, j_1) = f(i_2, j_2)$. Therefore, properties of remainders being accounted for, we have

$$\begin{cases} \operatorname{rem} \left(\frac{i_1 + \Delta i}{m} \right) = \operatorname{rem} \left(\frac{i_2 + \Delta i}{m} \right), \\ \operatorname{rem} \left(\frac{j_1 - 1 + \Delta j}{n} \right) = \operatorname{rem} \left(\frac{j_2 - 1 + \Delta j}{n} \right). \end{cases}$$

Multiplying the first equality by n and adding the second equality, we find

$$\begin{aligned} n \operatorname{rem} \left(\frac{i_1 + \Delta i}{m} \right) + \operatorname{rem} \left(\frac{j_1 - 1 + \Delta j}{n} \right) = \\ = n \operatorname{rem} \left(\frac{i_2 + \Delta i}{m} \right) + \operatorname{rem} \left(\frac{j_2 - 1 + \Delta j}{n} \right) \end{aligned}$$

i.e. $f(i_1 + \Delta i, j_1 + \Delta j) = f(i_2 + \Delta i, j_2 + \Delta j)$, q.e.d.

It is easy to show that above address indicator is optimal. Indeed, from (7) it follows $f(A) \leq mn - 1$. Further according to property i) the equality

$$\operatorname{card} f(\{1, 2, \dots, m\} \times \{1, 2, \dots, n\}) = mn$$

holds and since

$$\operatorname{card} f(A) \geq \operatorname{card} f(\{1, 2, \dots, m\} \times \{1, 2, \dots, n\})$$

taking into account zeroth page we find $\operatorname{card} f(A) = mn$.

Note that different problems require different subframe sizes. Therefore, rational compromise between relations of linear sizes of subframes and number of storage blocks can provide decreasing of time overhead for considerable number of image digital processing problems.

Let us agree to say that point-to-set map, satisfying (1) - (3), for which $\exists a_{ij}, a_{pq} \in \Omega$, and $F_\beta(a_{ij}) = F_\beta(a_{pq})$, is a quasipage scan. It is obvious that a quasipage scan F_β is optimal if

$$\begin{aligned} \forall \Omega \subset A, \forall F_\beta(a_{ij}) \operatorname{card} \{ \Omega \cap F_\beta(a_{ij}) \} \leq \\ \leq \operatorname{Int} \{ d/\delta + (d-1)/\delta \}, \end{aligned} \quad (8)$$

where $\delta = \operatorname{card} \{ F_\beta(a_{ij}) \}_{a_{ij} \in \Omega}, d = \operatorname{card} \Omega$.

Similarly, a map $\varphi : A \rightarrow \mathfrak{R}^\tau$, satisfying i), is an address quasiindicator if

$$\begin{aligned} \forall \xi \in \varphi(\{1, 2, \dots, m\} \times \{1, 2, \dots, n\}) \\ \operatorname{card} \varphi^{-1} \{ \xi \} \leq \operatorname{Int} \{ d/\delta + (d-1)/\delta \}. \end{aligned}$$

It should be noted that a quasiindicator is more general readdressing model than an indicator (property ii) is weakened) and address quasiindicator generates quasipage scan. Moreover, their implementation is based on repeated use of page scans. Hence required point-to-set map is diagonal product

$$F_\beta \Delta \Phi_\beta : A \rightarrow \mathfrak{R}^\tau \times \{1, 2, \dots, \operatorname{Int} \{ d/\delta + (d-1)/\delta \}\}$$

for which equality $\operatorname{card}(F_\beta \Delta \Phi_\beta(a_{ij}) \cap \Omega) = 1$ holds.

Taking into account that in applications, as a rule, $N = 2^{s'}$, $M = 2^{s''}$, $n = 2^{s'}$, $m = 2^{s''}$ it is possible to find quasipage scans such that

$$d/\delta = \arg \min_{\beta \in D(\beta)} \operatorname{card} \{ \Omega \cap F_\beta(a_{ij}) \}.$$

In other words, each memory block contains the same quantity of pixels for arbitrary subframe.

III. GENERAL RELATIONSHIPS FOR SUBFRAME PROCESSING

To find general relationships for producing regular page scans, first let us analyse column subframe displacement in horizontal direction. Consider each column of an arbitrary subframe from address subspace $\{1, 2, \dots, m\} \times \{1, 2, \dots, n\}$ and connect with it the set of permutation of order m :

$$\pi_j = \begin{pmatrix} 1, & 2, & \dots, & m \\ s_1, & s_2, & \dots, & s_m \end{pmatrix}$$

where meaning of s_h , $h \in \{1, 2, \dots, m\}$ lies in a number of memory pages. Moreover, let us consider a cyclic permutation $\pi_c = \begin{pmatrix} 1, & 2, & \dots, & m-1, & m \\ 2, & 3, & \dots, & m, & 1 \end{pmatrix}$. It is clear that

permutation $\pi = \pi_c^\alpha \pi_j$ allows to get new columns from all other subframes. Here the degree α means quantity of shift cycles ($\alpha > 0$ up, $\alpha < 0$ down). Notice that there exists only the identity permutation of lines under displacement in vertical direction which satisfies (1) – (4) and saves the translation invariant relation of equality. Hence, the total quantity of regular page scans, up to $(mn)!$, is

$$K = n^{\text{sign}(M-m)} + m^{\text{sign}(N-n)} - \text{sign}(M-m)\text{sign}(N-n)$$

and, consequently, $K \leq n + m - 1$.

From foregoing we are led to conclusion that general relationships for producing of regular page scans, up to bijection on $\{1, 2, \dots, m\} \times \{1, 2, \dots, n\}$, are written in the forms

$$F(a_{ij}) = n(\overline{\text{rem}}\{[i + \alpha_0([j/n] - 1)]/m\} - 1) + \overline{\text{rem}}\{j/n\},$$

$$F(a_{ij}) = m(\overline{\text{rem}}\{[j + \alpha_0([i/m] - 1)]/n\} - 1) + \overline{\text{rem}}\{j/m\},$$

$$\text{where } \overline{\text{rem}}\{p/r\} = \begin{cases} \text{rem}\{p/r\}, & \text{if } \exists \omega \in \mathbb{N} : p \neq \omega r, \\ r & \text{if } \exists \omega \in \mathbb{N} : p = \omega r, \end{cases}$$

« $\overline{\text{rem}}$ » denotes rounding up integer values; \mathbb{N} is set of positive integer numbers. It should be noted that $\alpha_0([i/n] - 1) = \alpha$ is value from the basic permutation π given above.

Finally, we can indicate subframe processing in terms of page scans (point-to-set maps satisfying presented results).

Let $\{\Omega_\lambda\}_{\lambda \in \Lambda}$ be a set of subframes to be treated, where $\lambda \in \mathbb{R}^2$ is any vector determining position of subframe in address space A , $\Lambda = \{\lambda_u\}_{u \in U}$, $\text{card } \Lambda = \text{card } U$, U is an index set. Then formula $\times_{u \in U} F_\beta\{\Omega_{\lambda_u}\}$, where « \times » denotes Cartesian product of page scans

$$F_\beta : \Omega_{\lambda_u} \rightarrow \Pi(A),$$

describes optimal sequential subframe treatment of 2-D data. In the cases when inequality (8) is valid quasioptimal sequential-subframe processing is given in following

form $\times_{u \in U} \times_{v \in V} F_\beta(\Psi(\Omega_{\lambda_u}^v))$, where map

$$\Psi : \{\Omega_\lambda\}_{\lambda \in \Lambda} \rightarrow \Pi(A)$$

is optimal page scan for subframe chosen viz.

$$\Omega_{\lambda_u} = \bigcup_{v \in V} \Omega_{\lambda_u}^v, \quad V = \{1, 2, \dots, \text{Int}(d/\delta) + (d-1)/\delta\},$$

$$\forall v_1, v_2 \in V \quad \Omega_{\lambda_u}^{v_1} \cap \Omega_{\lambda_u}^{v_2} = \emptyset,$$

$$\forall v \in V \quad \text{card } F_\beta\{\Omega_{\lambda_u}^v\} = \text{card } \Omega_{\lambda_u}^v.$$

Parallel subframe processing we can indicate as a set

$$\left\{ \Phi_\gamma \left(\left\{ \Omega_{\lambda_u} \right\}_{u \in U} \right) \right\}_7$$

of point-to-set maps (here $\gamma = \overline{1}, \overline{\Gamma}$; $\Gamma = \text{card } \{\Omega_{\lambda_u}\}_{u \in U}$) for which

$$\forall \gamma \in \{1, 2, \dots, \Gamma\} \quad \Phi_\gamma : \bigcup_{\lambda_u} (\Omega_{\lambda_u}) \rightarrow \Pi(A),$$

$$\Phi_\gamma \left(\bigcup_{\lambda_u} \Omega_{\lambda_u} \right) = \bigcup_{\lambda_u} \Phi_\gamma(\Omega_{\lambda_u}), \quad (9)$$

$$\forall \gamma \in \{1, 2, \dots, \Gamma\} \quad \text{card } \Phi_\gamma\{\Omega_{\lambda_u}\}_{u \in U} =$$

$$= \text{card } \{\Omega_{\lambda_u}\}_{u \in U}, \quad \forall \gamma, \forall \lambda \in \Lambda \quad \Phi_\gamma(\Omega_\lambda) = P_q \in \Omega_q, \quad (10)$$

$$\forall \gamma \in \{1, 2, \dots, \Gamma\}, \forall u_1, u_2 \in U : u_1 \neq u_2 \Rightarrow$$

$$\Rightarrow F_\beta(\Phi_\gamma(\Omega_{\lambda_{u_1}})) \neq F_\beta(\Phi_\gamma(\Omega_{\lambda_{u_2}})), \quad (11)$$

$$\forall \lambda \in \Lambda, \forall \gamma_1, \gamma_2 \in \{1, 2, \dots, \Gamma\} : \gamma_1 \neq \gamma_2 \\ \Rightarrow \Phi_{\gamma_1}(\Omega_\lambda) \neq \Phi_{\gamma_2}(\Omega_\lambda). \quad (12)$$

Now explain properties (9) – (12). Essence of relationship (9) consists in additiveness which, as a consequence, provides quasioptimal parallel-subframe processing. Formulae (10) describe structure of maps Φ_γ , namely, one element from given subframe is connected with this subframe. Implication (11) shows the choice of different storage pages for all subframes. Property (12) indicates action alteration of map Φ_γ when being passed to next set of elements to be processed.

CONCLUSION

Based on point-to-set maps models of conflictfree access to memory modules for parallel image processing have been proposed. Page scans are introduced as a toolkit to be exploited to obtain various memory models. Presented results provide reasonable choice of hardware, software and brainware to be used in task oriented computers for real time mode. All kinds of subframe processing can be formulated in terms of page scans. Single-cycle access is provided for simultaneous processing of both all pixels belonging to a subframe and set of pixels which are chosen per one from several overlapping subframes. All results can be easily extended to multidimensional structures of data with purpose of subset parallel processing.

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