The Wavelet Method for Image Compression

Vitaly T. Belinsky, Victor Yu. Chekh

Design Office "Storm"
at National Technical University of Ukraine
"Kiev Polytechnic Institute"

16 Polytechnichna St. 252056 Kiev, Ukraine
Phone: [044] 274 0295, - 442 6077
Fax: [044] 274 1015, - 443 7925
e-mail: chekh@ntu-kpi.kiev.ua, chekh@kbi.kiev.ua
http://www.parallelworlds.com.ua

The novel technology is proposed for the lossless compression of digital raster images of natural origin. The technology includes two steps; first one is the wavelet transform of the original images; on the second one the transformed data is encoded with particular optimum coding to obtain maximum compression ratio, and, sure, both inverse operations to obtain the initial image. The proposed technology is fast and naturally parallel and also gives a good compression ratio.

Introduction

The problem of compression of digital raster images of natural origin is actual for wide spectrum of applications - image archiving, remote sensing and Earth observation and so forth, especially for real time applications such as telecommunication, Internet, on-board systems. The general methods for data compression based on optimum coding are well known and widely used, particularly, for image compression. It must be noted that the compression rate could be additionally increased for the particular data because the information that the data belongs to particular class is a priory information, which could be used for decreasing of the entropy of the initial data.

The idea of the technology is to apply the orthogonal transform to the initial image to be compressed to decrease the entropy; then any optimum coding will result to better compression rate. To make the coding lossless the inverse transform must restore each pixel of the initial image from the transformed one.

The orthogonal transforms such as DDFT, DDCT and KL are widely used for image compression but mostly for lossy algorithms. We use wavelet transform, which can be inverted with no losses.

Method Description

Wavelet analysis is the novel conception based on orthogonal signal transform. Just as the Fourier sine and cosine harmonics, wavelets are the simple signals which could be any initial signal decomposed to. In contrary to sine and cosine, wavelets are finite in time and non-uniform, i.e. they depend on the initial signal. It is effective for decomposing real finite signals, particularly one needs of 16 wavelets or 256 Fourier harmonics to decompose the same notched signal. Fig. 1 reflects the general scheme (so called "pyramid scheme" [1]) for obtaining of signal wavelet decomposition. Wavelets have been already used for signal filtering, pattern recognition, even for matrix analysis, and also for signal and image compression.

J. Shapiro [2] and later Amir Said and William R. Pearlman [3, 4] proposed to use the particular type of wavelet transform - **S-transform** (sequential) [4, 5] for the image compression.

The series of integer digits ***c[n]***, n=0,...,N/2-1, where N is an even number can be :

$$l[n]=\lfloor c[2*n] + c[2*n+1])/2 \rfloor,\ n=0,...,N/2-1$$
$$h[n]= c[2*n] - c[2*n+1],\ n=0,...,N/2-1 \qquad (1)$$

where $\lfloor . \rfloor$ corresponds to download truncation. The sequences ***l[n]*** (after low-pass filtering) and ***h[n]*** (after high-pass filtering) forms the S- transform of ***c[n]***. Since the sum and difference of two integers correspond to either two odd or two even integers, the truncation is used to remove the redundancy in the least significant bit. The division and download truncation can be done with a single bit-shift. If each initial digit is limited with a size (say, 1 byte for the digit) it must be noted than ***l[n]*** and ***h[n]*** **can not** use the memory space used for ***c[n]*** as far as ***h[n]*** needs of one additional bit.

The inverse S-transform looks as:

$$c[2*n]= l[n] + \lfloor (h[n]+1)/2 \rfloor,$$
$$c[2*n+1] = c[2*n] - h[n]. \qquad (2)$$

The main advantage of this representation [3] is that for zero-mean random sequences the average variance for ***h[n]*** is smaller then the variance for ***c[n]***; the average variance for l[n] is similar to the variance for c[n]. To increase the effect it is obvious to apply the same decomposition to ***l[n]***. By the way, it is the rule for S-transforming of 1-D signals.

The 2-D transformation can be done by applying the transformation (1) sequentially to the rows and columns of the image. The coefficients corresponding to ***ll*** are the mean of 2*2 pixel blocks, and they form another image with half the resolution. The same transformation is applied to these reduced resolution "mean images" to form the hierarchical pyramid [7], see fig. 2. Note that the ***ll*** part of image has the same number of bits as the initial one but another parts needs of one (***lh*** and ***hl***) or two (***hh***) maximum number of bits additionally.

To decrease more the first order entropy it is possible to use predictors, i.e. more complex linear filters then used for obtaining ***h*** and ***l***. Such a transform (S+P- transform, i.e. sequential+predictive) can result to a smaller entropy then the S-transform but for particular images only; moreover, one needs to find optimum prediction coefficients for the particular images. S+P transform is slower because it needs of more complex calculations.

After the obtaining of the transformed image it can be encoded with any entropy coding methods such as arithmetic [8] or Huffman coding.

The method [3] is fast: in contrary to another orthogonal transforms (DDFT, DDCT, KL and others) S-transform needs ***O(log₂ N)*** operations, where N is the volume of initial data.

DFFT and S-transform need of $O(\log_2 N)$ operations but both could be directly applied only to the images with the dimensions equal to power of two. It looks like the serious disadvantage of the method [3] as far as it can not be successfully used for any image. Another its disadvantage is that the "pixels" of transformed image have a greater size then the pixels of the initial image.

To overcome these disadvantages the authors of the present paper have developed the modified S-transform, which uses the initial symmetry of the (1). It makes possible to collect modified ***l[n]***and ***h[n]*** at the same space as the initial ***c[n]***. Moreover, decreasing of the maximum range of the transformed image has also result to the decreasing of its entropy. This makes possible to increase compression rate using the same entropy coding methods. It must be noted that the increasing of the compression rate in average overcomes the increasing using the S+P transform. It also makes possible to investigate S-transformed images with the standard tools for raster images (see Fig. 3).

We also used the symmetry of the (1) applied to the finite images and the special boundary conditions. Now modified S-transform can be applied to the image of any dimension (even, odd, different number of rows and columns) and any size of pixel (8 bit, 12 bit, 16 bit). This makes the described technology useful for wide range of applications.

Another symmetry of (1) makes us possible to accelerate the modified S-transform about 2 times (sic!) and use 1.5 times less operation memory.

Conclusion

The described image compression technology is already software implemented. Its current market niches are space imaging, medical imaging, publishing, etc. Tests and comparing of our method to the nearest competitors particularly with the method by Said and Pearlman shows its following advantages:

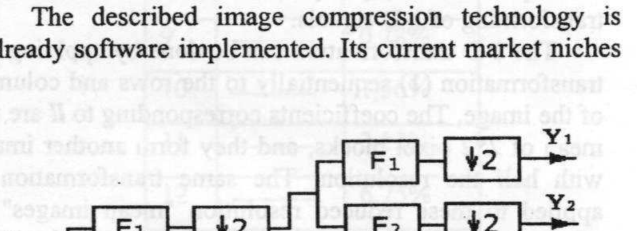
1. Better compression ratio (5-15% in average).
2. Faster encoding/decoding (1.5-2 times approximately)

It must be noted that the described technology is optimized namely for images of natural origin;

The embedded technology to be implemented on board of the Ukrainian Earth Observation satellite "Sich-2" (to be launched in 1999) for the compression of remote sensing data.

References

1. Adelson E. H., Simoncelli E., Hingorani R. Ortogonal pyramid transforms for image coding. Proc. SPIE, vol. 845, Cambridge, MA, pp. 50-88, Oct. 1987.
2. Shapiro J. M.. Embedded image coding using zerotrees of wavelets coefficients. IEEE Trans. Signal processing, vol. 41. Dec. 1993. pp. 3445-3462
3. Said A., Pearlman W. A. An Image Multiresolution Representation for Lossless and Lossy Compression. SPIE Symposium on Visual Communications and Image Processing, Cambridge, Ma, Nov. 1993.
4. Said A., Pearlman W. A. Reversible image compression via multiresolution representation and predictive coding. Proc. SPIE vol. 2094: Visual Commun. And Image Processing, Nov. 1993, pp. 664-674.
5. Ranganath S., Blume H. Hierarchical image decomposition and filtering using the S-transform. Proc. SPIE – Medical Imaging II, vol. 914, 1988, pp. 799-814,.
6. Roos P., Viergever A., MCA. Van Dijke, Reversible intraframe compression of medical images. IEEE Trans. Med. Imaging, vol. 7, Sept. 1988, pp. 328-336,.
7. Woods J. W., ed. Subband Image Coding. Kluver Academic Publishers, Boston, MA, 1991.
8. Witten I. H., Witten R. M., Neal R. M., Cleary J. G., Arithmetic coding for data compression. Commun. ACM, vol. 30, June 1987, pp. 520-540.



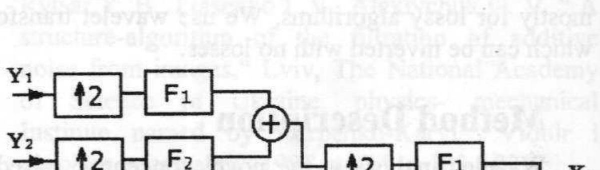


Fig. 1 Subband wavelet transform: direct **a)** and inverse **b)** Here X_n is the input signal, Y_j are the corresponding wavelets, F_j are the subband filters, ↓2 and ↑2 are down- and up-sampling, correspondingly.

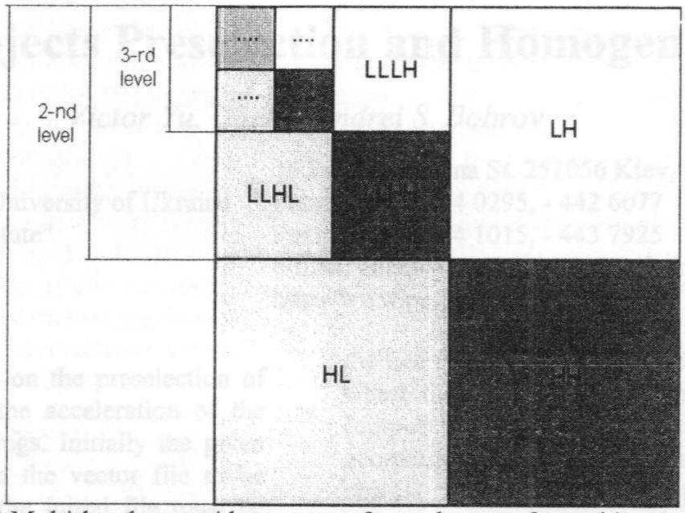
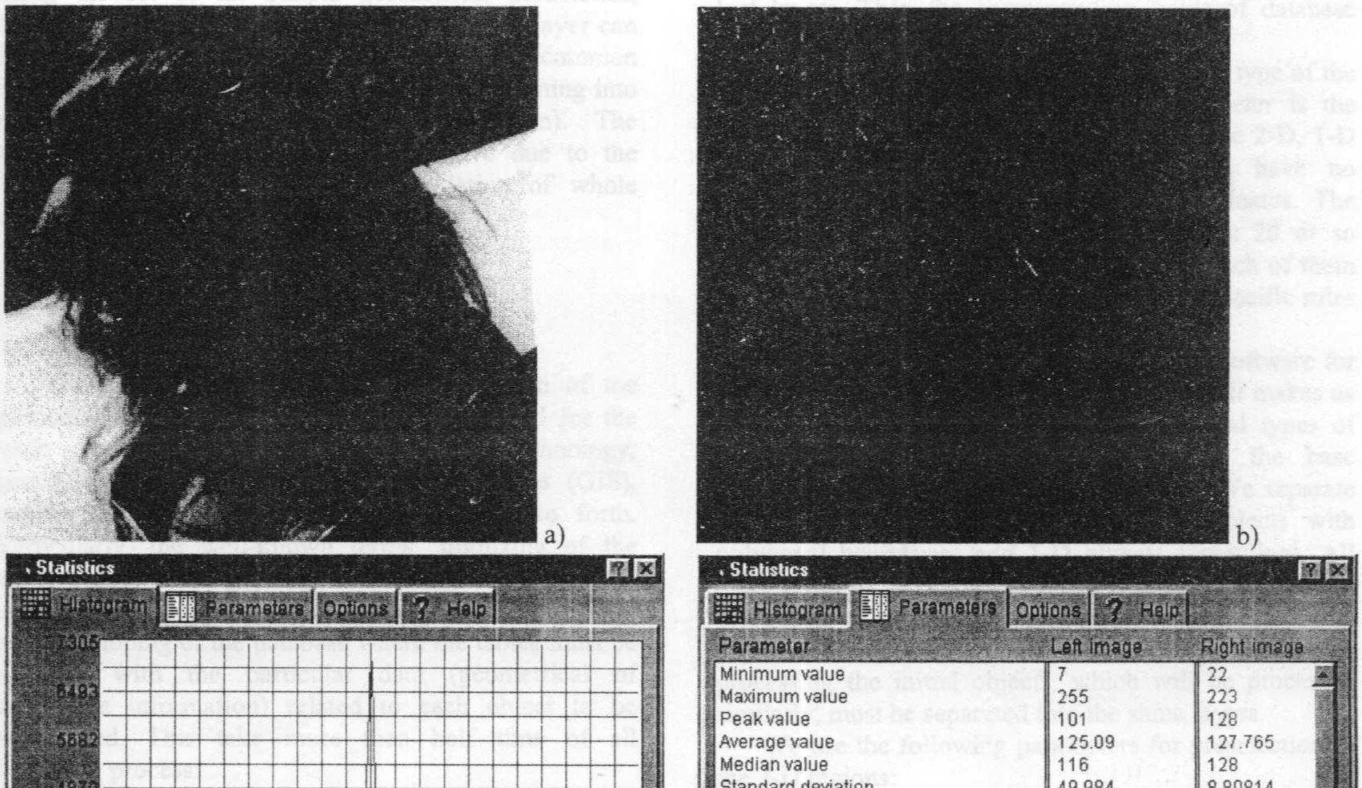


Fig. 2 Multi-level pyramid structure of wavelet-transformed image



Parameter	Left image	Right image
Minimum value	7	22
Maximum value	255	223
Peak value	101	128
Average value	125.09	127.765
Median value	116	128
Standard deviation	49.984	8.80814
Dispersion	2498.4	77.5834
Relative dispersion	0.159668	0.00475275
Entropy	7.3186	4.79727
Redundancy	1.6414	3.86094
Excess	-0.0572958	12.9237
Non-symmetry coefficient	0.0944961	0.0824165
Non-symmetry relation	0.658612	-0.47971
Coefficient of median displacement	0.00145378	0.000208847
Coefficient of peak displacement	0.00385283	0.000208847
Coefficient of neighbourhood belonging	1	1
Coefficient of neighbourhood belonging	1	1
Coefficient of neighbourhood belonging	1	1

Fig.3 Initial image a), it's modified wavelet transform b), histogram of brightness for both images (initial and transformed) and the statistical parameters (compare entropy and standard deviation)