Nonlinear adaptive filters for 1-D signal processing

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Abstract

The novel nonlinear adaptive filters for processing the signals corrupted with noise with nonstationary characteristics are proposed and investigated. They are based on iterative procedure assuming preliminary filtering, Z-parameter calculation and analysis with further adaptive selection of nonlinear filter parameters for local processing of signals. It is shown that the proposed adaptive approach provides a good trade-off of filter properties: noise suppression and spike removal ability combined with edge preservation property and low level of residual dynamic errors.

1. INTRODUCTION

The sampled information processes or sequences of measurements U(t) observed and registered in practice are usually the nonstationary ones containing step and ramp edges and other kinds of fragments like extrema, constant level and increasing/decreasing parts, etc. Besides, the noise or the measurement errors can also have statistical characteristics varying in time i.e. be non-stationary and non-gaussian. In such conditions the known linear filters fail to perform well and only nonlinear filtering algorithms can be applied [1]. However, the selection of a proper nonlinear filter is a problem as well because its properties depend upon many factors: filter type and scanning window length. Moreover, the statistical and, especially dynamical properties of nonlinear filters are analysed not thoroughly enough and they often can not be described analytically [1,2]. This is the reason of problems arising in forecasting of the output signal behaviour in cases of nonlinear filter application to noisy signals. So, first, the analysis of several typical nonliner filter dynamic and statistical characteristics is done. They are compared to the corresponding characteristics of mean filter for gaussian noise and the approximate dependences are obtained.

The nonstationary characteristics of noise and peculiar behaviour of information signal lead to expedience of adaptive nonlinear filtering. The basic idea is the local analysis of observed noise/signal mixture properties and respective selection of nonlinear filter type in order to ensure desired output characteristics. Several methods have been analysed and the corresponding recommendations concerning filter local selection have been put forward. The designed filters are, thus, locally-adaptive nonliner ones. They are based on Z-parameter and its modifications. The efficiency of proposed techniques and algorithms is confirmed by numerical simulation results.

2. SIGNAL/NOISE MODEL

The model of test signal S(t) included different fragments: constatnt level signal, increasing/decreasing parts, neighborhoods of extrema, step edge neighborhoods of these fragment junction. It is shown in Fig. 1,a. Every fragment contained 50 samples and it was possible to analyse the filter output characteristics separately for each fragment or to get integral characteristics for any part.

It was supposed that the noise may contain the additive (gaussian with zero mean and constant variance), multiplicative (with mean equal to unit and signal-dependent variance) and impulsive components. For this model the quantitative criterions both integral and local ones have been introduced.

3. NONLINEAR FILTER PROPERTY ANALYSIS

It was shown that for considered nonlinear filters (standard median, Wilcoxon, Hodges-Lehman and α--trimmed ones) the generalized expressions for describing dynamical and statistical characteristics could be got and they are derived using the expressions for standard mean filter with the same scanning window length. In particular, the dynamic errors of filter outputs can be written as

$$\Delta^{W} \approx 0.9 \cdot \Delta^{1} \quad \Delta^{H} \approx 0.85 \cdot \Delta^{1}, \ \Delta^{\alpha} \approx 0.6 \cdot \Delta^{1}, \tag{1}$$

where Δ^{W}, Δ^{H}, Δ^{α}, Δ^{1} are the dynamical errors of Wilcoxon, Hodges-Lehman, α-trimmed and mean filters, respectively.

The statistical characteristics of filter outputs can be approximately described in the following way

$$\sigma_W^2 \approx \sigma_H^2 \approx \frac{1.2\sigma_n^2}{N} = 1.2\sigma_l^2 \tag{2}$$

$$\sigma_\alpha^2 \approx \frac{1.35\sigma_n^2}{N} = 1.35\sigma_l^2, \tag{3}$$

where σ^2_W, σ^2_H, σ^2_α, σ^2_1 are the output signal residual fluctuation variances for Wilcoxon, Hodges-Lehman, α-trimmed and standard mean filters with equal scanning window sizes N, respectively.

The exceptional situation is the presence of spike in the scanning window current position. Its influence on output signal statistical characteristics has been analysed numerically for different filter types, scanning window sizes, noise variances and kinds of signals (in particular, its slopes). It was shown that the minimal integral output errors in case of spikes are provided by standard median filter with small scanning window size. That is why it can be recommended to be used in spike neighborhoods for locally adaptive filters. However, even better decision is the detection of spikes and their elimination from further "consideration" before processing. The main drawback of the standard median filter is its very poor performance for rapidly increasing/decreasing fragments of signal [2].

4. PROPOSED ADAPTIVE NONLINEAR FILTERS

The proposed adaptive filters assume the calculation of intermediate output using nonadaptive nonlinear filter (for example, the α-trimmed one with middle size N_p of scanning window). Then one or two Z-parameters are to be derived as

$$Z_i = \sum_{j=i-\frac{N_p-1}{2}}^{i+\frac{N_p-1}{2}} (U_j^f - U_j) \Bigg/ \sum_{j=i-\frac{N_p-1}{2}}^{i+\frac{N_p-1}{2}} \left|U_j^f - U_j\right|, \quad (4)$$

$$Z_i^s = \sum_{j=i-\frac{N_p-1}{2}}^{i+\frac{N_p-1}{2}} sign\,(U_j^f - U_j) / N_p,$$

$$sign\,(x) = \begin{cases} 1, & if\ x > 0, \\ 0, & if\ x = 0, \\ -1, & if\ x < 0, \end{cases} \quad (5)$$

where U_i^f denotes the preliminary filter output signal, U_i is the initial (input noisy signal), j and i are the indices.

The adaptive filtering can be performed on basis of separate and joint analysis of parameters Z and Z^s. Besides, it is possible to make smoothing of Z-parameter and its additional analysis using quasirange Q_Z. Thus, six variants of nonlinear adaptive filters was proposed and considered. It was shown that the addition of Q_Z analysis permits to detect edges while joint analysis of parameters Z and Z^s let to detect spikes. This "analysis" assumes the comparison of the current values of Z, Z^s and Q_Z with predetermined thresholds and undertaking decision on filter type and parameter selection on basis of several unequations and decision strategy. The best numerical simulation results have been obtained for variant of adaptive filtering performing joint analysis of Z, Z^s and Q_Z. For all considered variants of additive, multiplicative and impulsive noise characteristics it provided the best or very close to the best integral MSE in comparison to nonadaptive counterparts used as components of adaptive algorithm (standard median with N=5 and α-trimmed filters with N=9 and 13). The numerical simulation results (Table data) for different noise situations are presented in the report confirming this conclusion.

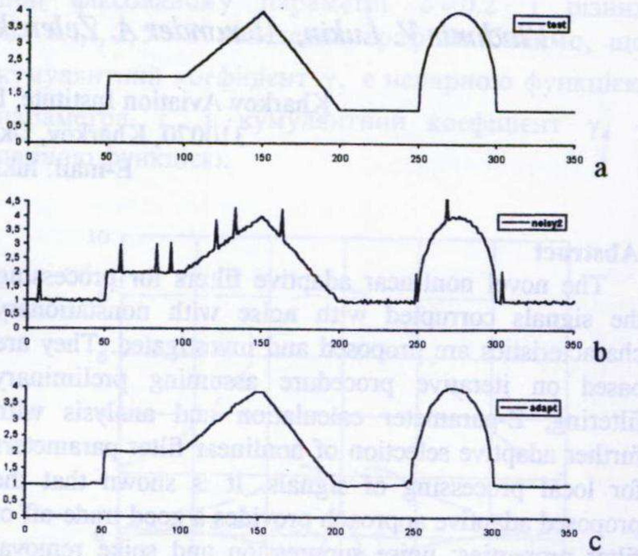

Fig.1. Simulation results: a)test signal, b)noisy test signal, c)result of processing by adaptive nonlinear filter

The simulations were performed for wide set of noise characteristics: additive noise variance was from 0.001 to 0.01, probability of spikes P varied from 0.0 to 0.03 and for multiplicative noise relative variance $\sigma_\mu^2 = k_o(S(t))^2$ parameter k_o was within the limits [0.001;0.01]. For case presented in Fig.1 (P=0.02, $\sigma_a^2 = 0.001$, $\sigma_\mu^2 = k_o(S(t))^2$, $k_o = 0.001$) the MSE value for standard median filter 0.0031, for α-trimmed (N=9) we got MSE=0.0042, for α-trimmed with N=13 MSE=0.0060 and for the best adaptive filter it was 0.0023. Local parameters of adaptive filter performance can be a little bit worse than for the best component filter because of errors in "situation recognition" based on Z-parameter analysis.

CONCLUSIONS

It is shown that an adaptive approach and algorithms of nonlinear filtering based on Z-parameter and proposed iterative procedure are rather simple and effective for wide variety of practical situations when the statistical characteristics of noise and the properties of signals are a priori unknown or one has restricted information about them. Their main adavantage is that they perform well for various types of fragments due to local adaptivity, thus ensuring both good integral and local characteristics (MSE and MAE).

REFERENCES

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