# An Axiomatic Definition of Knowledge in Uncertainty

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An axiomatic approach to the knowledge definition in uncertainty is suggested. The compound event certainty theorem and the inductive inference theorem are presented as well.

## 1. INTRODUCTION

The reliability and effectiveness of the modern automated system control and decision support system is determined by the ability to operate in the condition of initial information deficiency and distortion. Therefore the knowledge uncertainty problem draws attention of the various investigators[1-3].

## 2. FUNDAMENTALS

In general we can consider the knowledge about any arbitrary system as a set of pairs "input-output"

$$S = S_X \times S_Y = \{\langle x_i, y_i \rangle\},$$

mapping into some partially ordered set Z, i.e. as

$$\{\langle\langle x_i, y_j\rangle, z\rangle\}.$$

Value $z$ induces a preference relation on the set of events S. If $Z$ is the set of real numbers that mapping $T\colon\ S \to Z$ is a measure [4].

Let's introduce unnormalized countably additive trustworthy measure $u(s_i)$ of some event $s_i$ from the universal set of ones S, satisfying the following axioms:

1) Usual axioms of countably additive measure [5]:

$$u(\varnothing) = 0;$$

$$u(s_i) = w, \quad w \in [0, +\infty);$$

$$\forall s_i, s_j \in S\colon\ s_i \cap s_j = \varnothing,$$

$$u(s_i \cup s_j) = u(s_i) + u(s_j).$$

where trustworthy measure induces the preference relation on the set of events S:

$$\forall s_i, s_j \in S \quad u(s_i) > u(s_j) \Leftrightarrow s_i \succ s_j.$$

Having defined $s_k \subseteq S$ we simultaneously denote event $\overline{s_k} = S \setminus s_k$. So, chracterizing any event or phenomenon we always specify the trustworthy measure distribution

$$Q(S) = \{u(s_k)\}, \quad \forall k\colon\ s_k \in S.$$

Unlike to probability trustworthy is undetermined value, specified by trustworthy measure distribution validity level.

2) Validity level is a countably additive function:

$$V(\varnothing) = 0;$$

$$V(Q_i) = q, \quad q \in [0, +\infty);$$

$$\forall Q_i, Q_j \in G\colon Q_i \cap Q_j = \varnothing,$$

$$V(Q_i \cup Q_j) = V(Q_i) + V(Q_j).$$

Validity level induces the preference relation on the set of trustworthy measure distributions G:

$$\forall Q_i, Q_j \in G\colon\ V(Q_i) > V(Q_j) \Leftrightarrow Q_i \succ Q_j.$$

Call as event trustworthy the couple

$$\tau(s_i) = \langle u(s_i), q \rangle.$$

Let's call as trustworthy distribution T(S) the set of couples

$$\{\langle u(s_i), q\rangle\} = \langle Q(S), V(Q)\rangle.$$

To represent a complex event trustworthy define also axiom of consequence (3) and axiom of minimal validity (4):

3) Consequence axiom :

$$\forall T_i, T_j: \quad \forall k \quad u_i(s_k) = \alpha * u_j(s_k),$$

$$\alpha = \text{const}, \quad \alpha \geq 1,$$

$$F(T_i) \Rightarrow F(T_j),$$

where F is arbitrary functional on T.

4) Validity level of complex event is not more then minimum one for events its forming

$$\forall s_i, s_j, s_k: \quad \tau(s_i) = \langle u_1(s_i), q_1 \rangle,$$

$$\tau(s_j) = \langle u_2(s_j), q_2 \rangle,$$

$$s_k = s_i \oplus s_j,$$

$$\tau(s_k) = \langle u(s_k), q_3 \rangle:$$

$$u(s_k) = f(u_1(s_i), u_2(s_j), q_1, q_2),$$

$$q_3 = \min(q_1, q_2),$$

where $f$ – some function,

$\oplus$ – arbitrary operation.

All of possible couples $\tau(s_i)$ forms the cone within some Euclidean space - the Possible Events Space (PES) $\Omega^+$. According to the axioms no operation take out from the PES. Having fixed a metric on the $\Omega^+$, e.g. $\|\bullet\|_1$, we can define the equivalence relation, i.e. define a basis for classes clustering. By the usage of metric $\|\bullet\|_1$ they determine the tolerance relation as well, characterizing the different knowledge similarity in $\Omega^+$. The whole population of the PES points with the addition operation constitute the Abelian semigroup with zero (additive monoid).

Then introduce the axioms of knowledge inductive inference.

5) Possibility measure is determined by the set of examination:

$$P(S) = I(O);$$

$$I(\varnothing) = \varnothing;$$

$$I(O_t) = O_t;$$

where $O_t$ – the object state at moment t, $O_t \in (S \times R_+)$;

$O$ – set $\{O_t\}$ of the objects states, $O \subset (S \times R_+)$;

We close the fundamentals by two important statements, formulated in the form of existence and uniqueness theorems.

Theorem 1.

There is a unique compound event trustworthy function (measure), determined by the trustworthy of it components, that satisfy axioms (1)-(4).

$$\tau(A \cap B) = \xi(\tau(A), \tau(B)) = \xi\left(\langle u(A), q_A \rangle, \langle u(B), q_B \rangle\right) = \left\langle \min(q_A, q_B) \cdot \frac{u(A)}{q_A} \cdot \frac{u(B)}{q_B}, \min(q_A, q_B) \right\rangle$$

Note. In fact the theorem is the corollary from the product measure existence and uniqueness theorem [5].

Theorem 2.

There is a unique function f of the knowledge inductive inference that satisfy axioms (1)-(5):

$$f(u_1(A_i), u_2(A_i)) = u_1(A_i) + u_2(A_i).$$

# 3. CONCLUSION

Thereby, on the base of unnormalized countably additive measure the facilities of knowledge representation in uncertainty are discussed. Cardinal principles of the indeterminancy axiomatic formalization are given, difference from the known ones by the possibility to define the vagueness of the uncertainty itself. And, finally, two theorems of knowledge transformation are also presented.

# REFERENCES

1."*Нечеткие множества и теория возможностей. Последние достижения*". Под ред. Р.Р.Ягера//Пер. с англ.- М.:Радио и связь,1986. – 408 С.

2.Schalkoff, Robert J. "*Artificial intelligence : an engineering approach*".–McGraw Hill, Inc., 1990.–646P.

3."*Нечеткие множества в моделях управления и искусственного интеллекта*" // Под ред. Д.А.Поспелова. М.Наука, 1986.- 312 с.

4. Резниченко О.В. "*Формальный подход к определению знаний*". / Информационные системы. Сборник научных трудов, выпуск 2(10). НАНУ, ПАНИ, ХВУ - Харьков,1998.-стр.132-137.

5. Cohn, Donald L. "*Measure theory*". Boston, Basel, Stuttgart: Birkhausen, 1980. – 373 P.