

INFORMATION CRITERION FOR OPTICAL QUALITY ASSESSMENT HARDWARE WITH A HELP OF NOISED IMAGE OPTIMAL PROCESSING

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An information criterion is proposed for comparison of different optical systems and subsequent hardware intended for measurement of Optical Transfer Functions. The criterion permits to assess efficiency of a measuring system which is analyzing the image of the one-, two-, or three-dimensional random object described by an autocorrelation function. With a help of the criterion, a comparison has been fulfilled between different systems designed for optical image quality assessment of camera lenses. Hardware is based on optimal processing of noised image of test objects such as line, bright bar of certain width or straitforward boundary between bright and dark place.

errors in result of "exact" inversion of an integral transformation. Measuring systems of type b) are frequently better in result accuracy than those of type a). For example, systems like Fourier spectrometers are more effective especially in infrared region than common spectrometers.

In this paper we are using an united informational criterion for quality assessment of both systems of types a) and b). The criterion is based on ideology of an information theory and can be applied to assessment of measuring system before the measuring process. This approach is especially useful for system assessment at stage of the system design. A base for the criterion is optimal algorithm for signal processing which has been published in papers [2], [3] and lately generalized on to multidimensional data in papers [4] and [5].

INTRODUCTION

Different linear measuring systems are modeling by integral transformation of measured values. Such approach is used in such branches of science and technique as tomography, common and Fourier spectroscopy, surveillance systems of remote sensing, optical systems. It is prominent that all measured data are inevitably containing of registration noise.

Measuring systems can be divided upon two types: a) systems providing resulting data which can be used without additional processing; b) systems providing resulting data with a certain accuracy only after some additional analogue or digital processing of registered values. There are different known criteria for comparison of optical space invariant imaging systems of type a) based on use of Optical Transfer Functions (OTF)[1]. In the case of systems of type b) the choice of appropriate criterion is more difficult. It is connected with the fact that in such systems an inversion of an integral transformation, i. e. solution of Fredholm integral equation of the first kind, is usually so called ill-posed problem. In this situation any negligible small errors in registered data can lead to uncontrollably big

1. CRITERION FOR COMPARATIVE INFORMATIONAL ANALYSIS

Linear measuring system is working according to Fredholm integral equation of the first kind

$$\int_{-S}^S z(\xi)K(x, \xi)d\xi = F(x), \quad |x| \leq |R|. \quad (1)$$

Here

$\xi = (\xi_1, \xi_2, \dots, \xi_N)$, $d\xi = d\xi_1 d\xi_2 \dots d\xi_N$,
 $S = (S_1, S_2, \dots, S_N)$, $x = (x_1, x_2, \dots, x_M)$,
 $dx = dx_1 dx_2 \dots dx_M$, $R = (R_1, R_2, \dots, R_M)$. N designates dimension of unknown function $z(\xi)$ and M designates dimension of registered data. Inequality of a type $|x| \leq |R|$ means that $|x_1| \leq |R_1|$, $|x_2| \leq |R_2|$, \dots , $|x_M| \leq |R_M|$.

A right part of an equation (1) is known approximately so that $F(x) = f(x) + \gamma(x)$, where $f(x)$ is accurate value of the right part and $\gamma(x)$ is error of

registration (noise). It is clear from a physical considerations that functions $z(\xi)$, $f(\mathbf{x})$, $F(\mathbf{x})$ and kernel $K(\mathbf{x}, \xi)$ of the integral transformation (1) are square integrated at $|\mathbf{x}| \leq |\mathbf{R}|$, $|\xi| \leq |\mathbf{S}|$. Let unknown function $z(\xi)$ is called as an "object" and registered data $F(\mathbf{x})$ as an "image".

A linear optimal algorithm of general type has been proposed in paper [4] for the solving of the equation (1). In the algorithm, in comparison with a Wiener algorithm, an eigenfunctions of the equation (1) are not necessary that is leading to the much easier application of the algorithm in practice. The algorithm uses decomposition of unknown function $z(\xi)$ not upon eigenfunctions of the equation (1) but upon Karhunen-Loeve functions which are defined by an autocorrelation function of a random object. Objects of interest are usually stationary and decomposition of unknown functions $z(\xi)$ upon Karhunen-Loeve functions consists of common trigonometric Fourier series of N -dimension.

As a criterion for comparative informational analysis of optical quality assessment hardware we suggest value of

$$I = \ln \frac{\rho^2}{\langle \|z\|^2 \rangle} \quad (2)$$

which is the mean quantity of information in an image

$F(\mathbf{x})$. Here $\rho^2 = \left\langle \int_{-S}^S |\Delta z(\xi)|^2 d\xi \right\rangle$ is mean squared

error of object determination, $z^2 = \sum_{p=1}^{\infty} \langle |c_p|^2 \rangle$.

Coefficients c_p and error ρ^2 can be determined approximately by formulae listed in paper [2].

2. APPLICATION OF AN INFORMATION CRITERION FOR OPTICAL QUALITY ASSESSMENT

A lot of methods exists for evaluation of optical and photo-optical systems. A very general approach for characterizing the performance of lenses is to use OTF which describes the degradation of contrast and the spatial phase shift as a function of spatial frequency. A common way for the measurement of the OTF is based on an analysis of an image of simple test objects. The

most reliable methods are realized with such test objects as line, bright bar of certain width or straightforward line boundary between bright and dark place[1].

An OTF measuring procedure can be reduced to integral equation (1). Let width of a bright bar is $2b$. An image of such bar by a lens with the point spread function $K(\xi, \eta)$ is

$$\int_{-\infty-b}^{\infty} \int_{-b}^b K(x-\xi, y-\eta) d\xi d\eta = F(x, y), \quad |x| < R, \quad |y| < \infty. \quad (3)$$

As it is seen from equation (3), the function $K(\xi, \eta)$ corresponds to both linearly and shift invariant (stationary, isoplanatic) lens imaging when the response to an impulsive input located at the point (ξ, η) is simply a shifted version of the impulse response (point spread function) $K(\xi, \eta)$; that is, $K(x-\xi, y-\eta)$.

OTF is determined as Fourier transform upon point spread function $K(\xi, \eta)$ according to formula

$$T(\omega_x, \omega_y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} K(\xi, \eta) \exp[-i(\omega_x \xi + \omega_y \eta)] d\xi d\eta. \quad (4)$$

One-dimensional OTF is section of $T(\omega_x, \omega_y)$ by plane $\omega_y = 0$; that is, the OTF $T(\omega)$ is simple Fourier transform

$$T(\omega) = \int_{-\infty}^{\infty} \exp(-i\omega\xi) d\xi \int_{-\infty}^{\infty} K(\xi, \eta) d\eta \quad (5)$$

One-dimensional analogue of equation (3) is

$$\int_{-b}^b K(x-\xi) d\xi = \int_{x-b}^{x+b} K(\xi) d\xi = F(x) \quad (6)$$

where

$$K(\xi) = \int_{-\infty}^{\infty} K(\xi, \eta) d\eta = \frac{1}{2\pi} \int_{-\infty}^{\infty} T(\omega) \exp(i\omega\xi) d\omega. \quad (7)$$

Inserting of the equation (7) to the equation (6) gives the following correlation between the OTF and an image $F(x)$ of bright bar by width $2b$:

MEAN QUANTITY OF INFORMATION

$$\int_{-\infty}^{\infty} T(\omega) d\omega \int_{x-b}^{x+b} \exp(i\omega\xi) d\xi = F(x) \quad (8)$$

and for the even OTF we have

$$\frac{1}{\pi} \int_{-\Omega}^{\Omega} T(\omega) \cos(\omega x) \frac{\sin(\omega b)}{\omega} d\omega = F(x). \quad (9)$$

In equation (9) we have changed infinite limits on to finite limits $(-\Omega, \Omega)$ because the OTF is equal zero out of region $(-\Omega, \Omega)$ that is defined by an aperture of the lens.

Analogous expressions can be received for informational analysis of optical quality assessment hardware upon image of line or straight boundary between bright and dark place. Using an image $J(x)$ of straight boundary between bright and dark place, so-called edge spread function, we receive

$$\frac{1}{2\pi} \int_{-\Omega}^{\Omega} T(\omega) \frac{\sin(\omega x)}{\omega} d\omega = J(x) - \frac{T(0)}{2}. \quad (10)$$

With a help of an image $F(x, b \rightarrow 0) = A(x)$ of straight infinitely narrow slit which is called as line spread function, we receive

$$\frac{1}{2\pi} \int_{-\infty}^{\infty} T(\omega) \exp(i\omega x) d\omega = A(x). \quad (11)$$

In case of symmetry of line spread function

$$\frac{1}{2\pi} \int_{-\infty}^{\infty} T(\omega) \cos(\omega x) d\omega = A(x). \quad (12)$$

The expressions (9) - (12) can be used for detailed comparison of these three methods of OTF determination corresponding to mentioned above information criterion.

3. COMPARISON OF DIFFERENT METHODS FOR LENSES OTF MEASUREMENT BY VALUE OF

The basic assumptions of the OTF concept are met by a test object which is scanned by means of a well defined aperture. The radiance distribution of the test object is denoted as the object function and the transmittance profile of the scanning aperture in the image plane is denoted as the scanning function. It is important to choose a combination of the test object and scanning function that gives data which can be easily transferred into OTF values. Only in the case of a sinusoidal test object of spatial frequency ω scanned by a narrow slit the OTF is determined directly after standard normalization to unity at $\omega = 0$. In practice, test objects with more simple shapes easier to produce than a sinusoidal shape are usually used.

The first of such methods for lenses OTF measurement with test object of simple shape uses a narrow slit by infinitely small width and fixed narrow slit as a scanning aperture. The OTF is then calculated as the Fourier transform of the line spread function. As it follows from an equation (5),

$$T(\omega) = \int_{-\infty}^{\infty} A(x) \exp(-i\omega x) dx \quad (13)$$

with account on the definition of line spread function

$$A(\xi) = \int_{-\infty}^{\infty} K(\xi, \eta) d\eta.$$

In the second method for lenses OTF measurement with test object of simple shape, a straight bright bar of variable width $2b$ is used as an object and fixed narrow slit as a scanning aperture. Actually, the scanning procedure is performed by changing the width of the tested bar. At this, the normalized irradiance $C(b)$ in the center of the bright bar is measured as a function of the tested bright bar and the OTF is calculated as the Fourier sine transform

$$T(\omega) = 1 - \frac{\omega}{2} \int_0^{\infty} [1 - C(b)] \sin\left(\frac{\omega}{2} b\right) db. \quad (14)$$

This equation is valid in case of symmetry of line spread function [1].

The third method for lenses OTF measurement with test object of simple shape uses straight boundary between bright and dark place as an object and fixed narrow slit as a scanning aperture. The OTF is then

calculated by means of the Fourier cosine transform of the edge spread function $J(x)$

$$T(\omega) = \omega \int_0^{\infty} [1 - J(x)] \cos(\omega x) dx. \quad (15)$$

This equation is also valid only in case of symmetry of a line spread function [1].

Comparison have been provided between values of the mean quantity of information for the measuring systems providing OTF data in general way and these of the same systems providing measuring data with the same error of experiment but only after additional digital optimal processing. measuring systems providing OTF data in general way have better resulting accuracy in OTF values in comparison with the same systems providing measuring data with the same error of experiment but only after additional digital optimal processing.

As a result, the measuring systems providing OTF data in general way have better resulting accuracy in OTF values in comparison with the same systems providing measuring data with the same error of experiment but only after additional digital optimal processing. A difference between the OTF values depends on number of sampling points in registered image and in appropriate difference in mean squared value of measurement error for different types of the measuring system.

The value of the mean quantity of information gives also possibility to assess different OTF measurement methods with using a test object by simple form. The value of the mean quantity of information have been calculated for all three methods of the lens OTF determination at the same signal-to-noise ratio equal to 10%. The value of the mean quantity of information gives preference to the methods with using a test object in form of bright bar of finite width or straitforward boundary between bright and dark place. Maximal values of the mean quantity of information are decreasing at decreasing of number of sampling points in registered image and at increasing of mean squared value of measurement error for different methods of the OTF measurement

CONCLUSION

An effective information criterion for optical quality assessment hardware for measurement of image quality is developed with account on possibilities for numerical processing of the image of the one-, two- or three-dimensional random object. It is based on the mean

quantity of information which is connected with a probability of a measurement error to exceed a given value. To calculate the mean quantity of information, it doesn't necessary to know eigenfunctions of a measuring system. This fact permits to apply proposed criterion for assessment of different imaging devices.

Due to concept of the mean quantity of information, image quality assessment systems have been compared in account for their ability to measure Optical Transfer Function. As a result, the measuring systems providing OTF data in general way have better resulting accuracy in OTF values in comparison with the same systems providing measuring data with the same error of experiment but only after additional digital optimal processing.

The same type of comparison has been made between different methods of the OTF measurement. A performance criterion based on the value of the mean quantity of information gives preference to the methods with using a test object in form of bright bar of finite width or straitforward boundary between bright and dark place.

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